This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

7/6

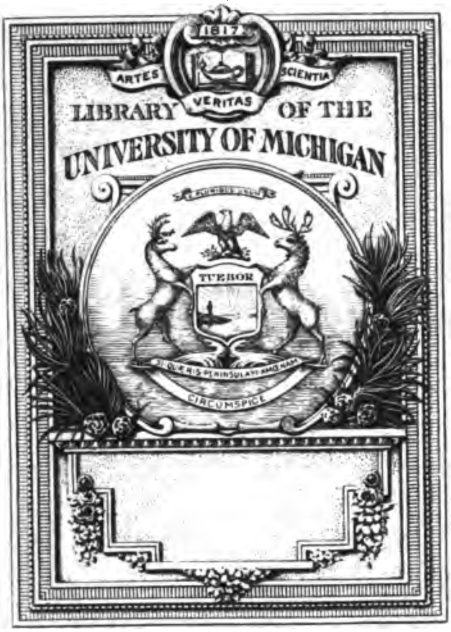
1817
ARTES
SCIENTIA
VERITAS
LIBRARY OF THE
UNIVERSITY OF MICHIGAN
E PLURIBUS UNUM
TUEBOR
SI QUAERIS PENINSULAM AMOENAM
CIRCUMSPICE

Cott 15.

E Horwood
Great Marlow
Bucks

QA
73
.M15
181

7/

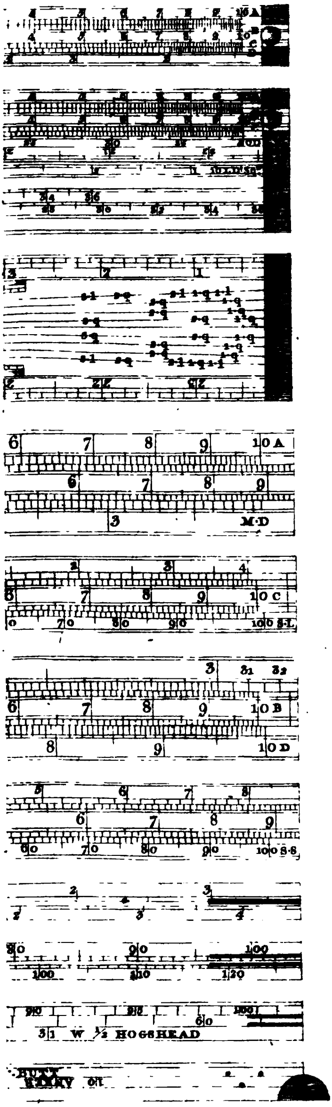


1817
ARTES VERITAS SCIENTIA
LIBRARY OF THE
UNIVERSITY OF MICHIGAN
E PLURIBUS UNUM
TUEBOR
SI QUAERIS PENINSULAM AMOENAM
CIRCUMSPICE

loc. 15.

E Horwood
Great Marlow
Bucks

Q
73
M
18



line of numbers.

Engraved by J. Mannier

THE
DESCRIPTION AND USE
OF THE
SLIDING RULE,
IN ARITHMETIC,
AND IN
THE MENSURATION OF SURFACES AND SOLIDS.

ALSO,

THE DESCRIPTION
OF THE
SHIP CARPENTER'S SLIDING RULE,
AND
ITS USE APPLIED TO THE CONSTRUCTION OF
MASTS, YARDS, &c.

TOGETHER WITH

THE DESCRIPTION AND USE
OF THE
GAUGING RULE, GAUGING ROD, & ULLAGE RULE.

By ANDREW MACKAY, LL.D. F.R.S. EDIN. &c.
MATHEMATICAL EXAMINER TO THE CORPORATION OF TRINITY-HOUSE,
THE HONOURABLE THE EAST INDIA COMPANY,
AND CHRIST'S HOSPITAL, LONDON.

THE SECOND EDITION,
Improved, Enlarged, and Illustrated with an accurate Engraving of the different Rules.

EDINBURGH:
PRINTED FOR OLIPHANT, WAUGH, AND INNES;
AND SOLD BY LONGMAN, HURST, REES, ORME, AND BROWN,
LONDON.

1811.

ENTERED IN STATIONERS-HALL,
According to Act of Parliament.

ADVERTISEMENT TO SECOND EDITION.

This Edition was corrected and printed by the Author previous to his death, and has been purchased from his Widow by the Publishers.

Edinburgh, 18*th Nov,* 1811.

ERRATA.

For Section II. page 99, *read* Section III.
For Section III. page 106, *read* Section IV.

Teape, Printer, London.

Hist. of science
Sanders
8-24-38
36908

PREFACE.

THE SLIDING RULE is an instrument of so extensive use, particularly in the mensuration of wood, stone, glass, painting, plastering, &c. that it ought to be in the hands of every artificer.

10-14-39 mell

In order, therefore, to assist those who may still be unacquainted with the nature and use of this instrument, the following concise and plain directions were drawn up; and which, it is presumed, will also be found of service to those who have some previous knowledge of this subject. In these directions, plainness and perspicuity have chiefly been aimed at, insomuch, that it is hoped they will be found sufficiently intelligible, even without the assistance of a teacher.

That this small Treatise may be farther useful to artificers of every denomination, another Sliding Rule employed by Ship-carpenters, and commonly known by the name of MUNGO MURRAY's Rule, is also described, and its use exemplified in determining the dimensions of masts and yards, and in the construction of them; and also in making ten and twelve square barrels for capstans. This last will also be found useful to joiners and others in making columns, &c.

To this edition, the demonstrations of

the methods of constructing the eight, ten, and twelve square lines are added: and in order to render it more generally useful, the description and use of the *gauging rule, diagonal rod,* and *ullage rule,* in their present improved state, are annexed.

As this tract was originally written for the express purpose of measuring wood, and for the construction of masts and yards, the former arrangement of the problems is, in general, preserved; and the additional problems, which will be found useful in the various departments of mensuration are annexed. It was thought unnecessary to add more problems, as, it is presumed there are a sufficient number for those for whose use this Tract is intended; and those omitted will serve as an exercise.—Although the several problems are, in general, performed by the sliding rule, yet it will tend greatly to improvement to perform them by arithmetic, and, if convenient, also by logarithms.

Algebraic characters, or symbols, although the shortest method of expressing the relations of magnitudes to each other, are carefully avoided, in order to prevent any apparent difficulty to those unacquainted with that branch of science.

LONDON,
17th OCTOBER, 1806.

CONTENTS.

CHAP. I.

Containing the Description and Use of the Sliding Rule.

PAGE

Sect. I. The Description of the Sliding Rule 1

II. The Use of the Sliding Rule in Arithmetic 7

III. Application of the Sliding Rule to the Mensuration of Surfaces 26

IV. The Sliding Rule applied to the Mensuration of Solids 43

CHAP. II.

Containing the Description and Use of the Ship Carpenters' Sliding Rule.

Sect. I. The Description of Lines upon one Side of the Rule 83

II. Of the Lines for ascertaining the Lengths and Diameters of Masts, Yards, &c. 94

III. The Description and Use of the Lines upon the second Side or Face of the Rule 99

IV. The Investigation of the Methods of constructing the Eight, Ten, and Twelve Square Lines ..106

CHAP. III.

Containing the Description of the Gauging Rule; and its Use in Gauging and Ullaging Casks, &c.

Sect. I. Introduction 113

II. Description of the Gauging Rule 114

III. The Method of finding the Divisors, Multipliers, and Gauge Points used in Gauging 118

IV. Use of the Rule in Cask Gauging 121

V. Use of the Gauging Rule in finding the Content of a Cistern, Couch, or Floor of Malt 126

CONTENTS.

CHAP. IV.

Containing the Description of the Diagonal or Gauging Rod: and its Use in finding the Content of Casks.

SECT. I. The Description of the Gauging Rod 129
II. The Manner of using the Gauging Rod 181

CHAP. V.

Containing the Use of the Ullage Rule.

SECT. I. The Description of the Ullage Rule 133
II. The Use of the Ullage Rule in ascertaining the Quantity of Liquor in a Cask 134

THE FOLLOWING ARE THE PRINCIPAL CHARACTERS USED IN THIS TRACT:

The sign +, called *plus*, when placed between two quantities, signifies that the second is to be added to the first.

The sign —, called *minus*, when placed between two numbers, signifies that the second is to be subtracted from the first.

The sign ×, placed between two quantities, denotes that the first is to be multiplied by the second.

The sign ÷, placed between two quantities, signifies that the first is to be divided by the second. This is also expressed by placing the first, or dividend above a line, and the divisor below it.

The sign =, called the *sign of equality*, signifies that the quantities, between which it is placed, are equal.

The characters :, : :, and : when placed between numbers, signify geometrical proportion: the first denotes the words *is to*, the second *as*, or *so is*, and the third *to*. Thus 3 : 5 : : 9 . 15; that is, 3 is to 5, as 9 to 15.

THE

DESCRIPTION & USE

OF THE

SLIDING RULE.

&c.

CHAP. I.

CONTAINING

THE DESCRIPTION OF THE SLIDING RULE,

AND

ITS USE APPLIED TO ARITHMETIC:

AND TO THE

MENSURATION OF SURFACES AND SOLIDS.

SECTION I.

The Description of the Sliding Rule.

THIS rule is formed of two pieces of wood, each a foot long, and connected by a joint: in one of these pieces is a slip of wood or brass, sliding in a groove; and hence the name of the rule.

The principal lines on this rule are those on that part in which is the slide. These are, three lines called *lines of numbers;* each containing two equal and similar sets of divisions;

and, therefore, called *double lines of numbers.* Two of these double lines of numbers are upon the slide, and marked B, and C, at the right-hand extremity; and the line above, is marked A. The line below the slide, is a single line of numbers, and, therefore, the interval between any two divisions on this line, is double the interval between the corresponding divisions on either of the lines A, B, or C. This line is marked D, and is called the *girt line,* from its use in finding the content of a tree, by the common method, by taking the girth of the tree with a line, one-fourth of which is usually taken for the side of the square. The girt line begins at 4 and ends at 40, in order that 12 may be near the middle of the line; and by this means, the content of a solid is, in general, found at one operation. Upon the other parts of the rule are various lines or tables, according to the fancy of the maker, or the use for which it is intended. Thus, on the same side of the rule in which is the slide, but on the other part, there is put on some rules, a table expressing the value of a load of wood containing 50 cubical feet, from 6d. to 2s. per solid foot. For example, at 6d. per foot, the value of the load is L.1, 5s.; at 6¼d. the value is L.1, 6s.; at 6½d. it is L.1, 7s. 1d.; at 6¾d. the value is L. 1, 8s. 1d. &c. Again, on other rules, there is a table exhibiting the value of an hundred weight, or 112lbs. from a farthing to a shilling per cwt. Thus at one farthing per lb. the cwt. will amount to L.0, 2s. 4d.; at 4¾d. per lb. the value of the cwt. is L.2, 4s. 4d. &c. Other rules, have a table containing the length of a rope or cable, of a given circumference, that will make an hundred weight. Thus, of a rope 6 inches in circumference, it will require 13 fathoms 3 feet to make a cwt. On other rules, are the proportional dimensions of masts, and yards, scales for squaring yards, capstans, &c.

Upon the other side of the rule, are scales of inches, and parts of an inch, or the sectoral lines, such as a line of equal parts, on each leg, usually called a *line of lines*; for laying down scales, performing the various operations in proportion, &c. Scales for the construction of masts, yards, bowsprits, &c. Upon this side also, of some rules, are the proportional dimensions of masts, and yards.—The length answering to any given breadth, to make a square foot. The length answering to any given thickness, to make a solid foot, &c.

One edge of the rule is commonly divided into 100 equal parts; and upon the other edge, two lines of numbers are sometimes placed; one being a single, and the other a triple line of numbers; which are useful in finding the cube root of any proposed number, and conversely.

OF THE LINE OF NUMBERS.

The line of numbers is divided into two equal parts; each of these parts is divided into ten unequal parts; and each of these parts is subdivided into ten or more parts.

If the 1 at the beginning of the line represents 1, the next primary division represents 2, the third 3, and so on; the 1 at the middle of the line represents 10, the next primary division is 20, and the 1 at the end of the line represents 100. But if the 1 at the beginning of the line denotes 10, then the 1 at the middle of the line represents 100, and the 1 at the end represents 1000; and the intermediate divisions are to be estimated accordingly. Hence, it may be observed, that the numbers in the second half of the line, are ten times the corresponding numbers in the first half; and hence the following, rule to find the place of any proposed number on the line, will be obvious.

Among the primary divisions, take the same as in the highest figure in the given number; among the intermediate principal divisions, take that which is the same as the second highest figure;

figure; and for the other figures, a proportional part of the space between the last found, and the next following of the principal intermediate divisions, is to be taken by estimation.

EXAMPLES.

I.

Required the point representing 49?

The primary division 4, on either half, is to be reckoned 40; and numbering forward to the right hand, until the 9th of the principal intermediate divisions, will be the point representing 49. This point may also represent 490, 4900, &c.

II.

Required the point representing 164?

The 1 at the beginning, or middle of the line, is to be esteemed 100. In this case, therefore, the next primary division 2 denotes 200. Now reckon forward from 100 among the principal intermediate divisions until the 6th, which point will, therefore, represent 160; and since the interval between 160 and 170 is divided into 5 parts, each of these parts is, therefore, to be estimated 2; and because the unit figure of the proposed number is 4, reckon forward two divisions; this last point will, therefore, be the place representing 164.

III.

The point on the line of numbers representing 543 is required?

The primary division 5 on the line, will represent 500; and of the intermediate principal divisions 4 will represent 540. Now the interval between 540 and 550 being supposed to be divided into ten parts, then the third division, taken by estimation, from 540, will be the point representing 543.

IV.

The place of 1798 on the line of numbers is required?

The 1 at the beginning or middle of the line, is to be taken as 1000, and reckoning forward to the 7th of the principal intermediate divisions, will be the place of 1700; the 4th of the subdivisions will be the place of 1780; the interval between which, and the next division 1800, is 20, of which the 18th division, being taken by estimation, will be the place representing 1798.

V.

Required the point on the line of numbers which represents 1806?

The 1 at the middle being taken to represent 1000, then the 8th intermediate principal division will be the place of 1800; and one of the subdivisions being 20, the point representing 6, of this interval, being taken by estimation, will be the place of 1806.

The line of numbers is a scale of logarithms, and is constructed as follows.

Make a scale of equal parts, and divide it as accurately as possible, and in such a manner, that 100 upon it, may be equal to the interval between 1 and 10, on the proposed line of numbers. Now, the logarithm of 2 is to be taken from this scale, and laid off from 1 on the line of numbers; then, the logarithm of 3, &c. and in this manner *the line of numbers* is graduated.

SECTION II.

The Use of the Sliding Rule in Arithmetic.

PROBLEM I.

Multiplication by the Sliding Rule.

Multiplication is the method of finding the product of two given numbers. These numbers are usually called *Factors;* but more frequently, the *Multiplicand*, and *Multiplier*.

RULE.

Set 1 on the line B, to either of the factors on A; then against the other factor of B, is the product on A.

EXAMPLES.

I.

Required the product of 3 by 7?

Set 1 on B, to 3 on A; then opposite to 7 on B is 21, the product on A.

II.

What is the product of 15 by 8?

Set 1 on B, to 8 on A; and opposite to 15 on B, is 120 on A, the product.

III.

Required the product of 24 by 18?

Set 1 on B, to 24 on A; then opposite to 18 on B, is 432 on A.

REMARK.

When the product extends to three figures, it often happens that the unit figure cannot be immediately distinguished on the rule: this, however, is easily obtained by multiplying the unit figures in the factors. Thus in the preceding example, 4 multiplied by 8 is 32; hence, the unit figure of the product must be 2.

PROBLEM II.

Division by the Sliding Rule.

Division is the method of finding the number of times one quantity or number is contained in another:—The first of these numbers is called the *Divisor*, the second is called the *Dividend*, and the resulting number is called the *Quotient*.

RULE.

Set the divisor on B, to the dividend on A; then against 1 on B, is the quotient on A.

EXAMPLES.

I.

Divide 63 by 7?

Set 7 on B, to 63 on A; then opposite to 1 on B, is 9 on A, the quotient.

II.

Divide 480 by 15?

Set 15 on B, to 480 on A; and opposite to 1 on B, is 32 on A, the quotient.

III.

III.

Divide 912 by 19?

Set 19 on B, to 912 on A; and opposite to 1 on B, is 48 on A.

IV.

Divide 1500 by 40?

Place 40 on B, to 1500 on A; and opposite to 1 on B, is the quotient $37\frac{1}{2}$ on A.

V.

Divide 5840 by 64?

Set 64 on B, to 5840 on A; and opposite to 1 on B, is the answer $91\frac{1}{4}$ on A.

REMARK.

In the same manner is a vulgar fraction reduced to a decimal; namely, by setting the denominator on B, to the numerator on A; then opposite to 1 on B, is the required decimal on A.

EXAMPLES.

I.

Reduce $\frac{4}{5}$ to a decimal fraction?

Set 5 on B, to 4 on A; then opposite to 1 on B, is .8 the decimal of $\frac{4}{5}$, on A.

II.

Reduce $\frac{3}{4}$ to a decimal fraction?

Set 4 on B, to 3 on A; and opposite to 1 on B, is .75 on A, the corresponding decimal fraction.

III.

Reduce $\frac{5}{8}$ to a decimal?

Set 8 on B, to 5 on A; and opposite to 1 on B, is .625 on A, the decimal required.

PROBLEM III.

Proportion, or the Rule of Three, by the Sliding Rule.

THE RULE OF PROPORTION, or as it is usually called, THE RULE OF THREE, or the GOLDEN RULE, is when three numbers are given to find a fourth, to which the third has the same proportion, that the first has to the second.

RULE.

In stating the questions in this rule, the first and second terms are to be of the same name, and the third term is to be of the same name as that required.

Let, therefore, that term which is of the same name as the required one, be placed as the third term of the proportion. Then consider, from the nature of the question, whether the required term is to be greater or less than the third term. If it is to be greater, the least of the other two terms is to be placed as the first term of the proportion, and the other as the middle term: but if it ought to be less, the greater of the two terms is to be placed as the first, and the least in the middle.

Then set the first term on B, to the second on A; and opposite to the third term on B, is the fourth term or answer on A.

EXAMPLES.

I.

What is the value of 15 yards of cloth, when 40 yards cost L.24?

As 40 : 15 : : 24 :

Now set 40 on B, to 15 on A; and opposite to L.24 on B, is L.9 on A, the answer.

II.

If 7 yards of muslin cost L.1, 3s. how many yards may be bought for L.2, 17s.?

As 23s. : 57s. : : 7 :

Now set 23 on B, to 57 on A; then opposite to 7 on B, is the number required, 19 on A.

III.

The debts of a bankrupt amounted to L.750, and his effects to L.400; how much will be recovered for a debt due by him of L.150?

As 750 : 150 : : 400:

Set 750 on B, to 150 on A; then opposite to 400 on B, is 80 on A, the answer.

IV.

If I lend a friend L.450 during 10 months, how long ought he to lend me L.300?

As 300 : 450 : : 10:

Now set 300 on B, to 450 on A; and opposite to 10 on B, is 15 months on A, the answer.

V.

Bought 40 yards of fine cloth for L.24, how may I sell it per yard to gain L.12 upon the whole?

L.24+12=L.36.

And 40 : 1 : : 36:

Set 40 on B, to 1 on A; then opposite to L.36 on B, is $\frac{9}{10}$ of a pound, or 18 shillings on A:

or opposite to 720 (the number of shillings in L.36) on B, is 18 shillings, the answer on A.

VI.

A traveller sets out from Edinburgh to London, the distance being 400 miles, and walks at the rate of 21 miles a day—three days after, another is sent after him, who travels at the rate of 30 miles a day. In what time will the second overtake the first; and how many miles will each have travelled?

The first was 63 miles, =21×3, from Edinburgh, when the second set out; and the second gained upon the first nine miles, =30—21, per day.

Therefore, set 9 on B, to 63 on A; and opposite to 1 on B, is 7, the number of days required by the second to overtake the first; and the number of miles travelled will be 210=30×7. or =63+21×7.

VII.

Two travellers set out at the same time, one from London to Edinburgh, and the other from Edinburgh to London, by the same road—the one from London travels at the rate of 23 miles daily, and the other from Edinburgh travels at the rate of 27 miles daily. In what time will they meet, and how many miles will each have travelled, the distance between the two cities being 400 miles?

As 23+27 or 50 : 400 : : 1 :

Now set 50 on B, to 400 on A; then opposite to 1 on B, is 8 on A, the number of days in which they will meet.

Again set 1 on B, to 8 on A; and opposite to 13 on B, is 184 on A, the distance travelled by the person from London. Hence the person from Edinburgh travelled 216 miles.

VIII.

Upon Wednesday morning at three o'clock, a ship, going at the rate of eight knots an hour, passed the island of May, upon a direct course for the Naze of Norway, the distance being 332 miles. It is required to know when she will make the Naze, the rate of sailing being supposed uniform?

Set 8 on B, to 332 on A; then opposite to 1 on B, is 41½ hours on A; which reckoned forward from Wednesday morning at 3 o'clock, will give half past eight on Thursday evening.

IX.

Required the interest of L.840 for 13 weeks, at 5 per cent?

As 52 : 13 : : 5 :

Set 52 on B, to 13 on A; and opposite to 5 on B, is 1¼ on A.

And as 100 : 840 : : 1¼ :

Now set 100 on B, to 840 A; then opposite to 1¼ on B, is the required interest 10½, or L.10, 10s. on A.

X.

A ship is purchased for L.1200, required the value of that person's share who holds three-eighths?

As 8 : 3 : : 1200 :

Set 8 on B, to 3 on A; then opposite to L.1200 on B, is L.450 on A, the required value.

XI.

If the breadth of a piece of wood is 9 inches, what length is necessary to make a square foot?

As 9 : 12 : : 12 :

Now set 9 on B, to 12 on A; and opposite to 12 on B, is 16 inches on A. Or set 9 on B, to 144 on A; and opposite to 1 on B, is the answer 16 inches on A.

XII.

If 30 men perform a piece of work in 18 days, in what time will 20 men do the same?

As 20 : 30 : : 18.

Set 20 on B, to 30 on A; then opposite to 18 on B, is the answer, 27 days, on A.

XIII.

A garrison of 1500 men has provisions for twelve months, how long will the provisions last, at the same allowance, if the garrison be reinforced by 500 men?

1500+500 = 2000 men?
As 2000 : 1500 : : 12 :

Set 2000 on B, to 1500 on A; then opposite to 12 on B, is 9, the answer in months on A.

XIV.

Required the Heliocentric latitude of Saturn, ⅌ M. de Lambre's Tables, answering to $0^{s}\ 16^{\circ}\ 17'\ 24''$ of argument?

Latitude to $0^{s}\ 16^{\circ} = 0^{\circ}\ 41'\ 19''$
to $0\ \ 17 = 0\ \ 43\ \ 49$

Difference······ $2\ \ 30 = 150''$

Now set 60 on B, to $17'\ 24''$, or 17. 4 on A; then opposite to $150''$ on B, is the proportional part $43\frac{1}{2}''$ on A; which added to $0^{\circ}\ 41'\ 19''$, the sum $0^{\circ}\ 42'\ 2\frac{1}{2}'$ is the latitude required.

EXAMPLES FOR EXERCISE.

I.

If 9 yards of cloth cost 3 guineas, what is the value of 20 yards? Ans. L.7.

II.

How many yards of muslin may be bought for L.14, 12s. when the price of 7 yards is L.1, 8s. Ans. 73 yards.

III.

If $7\frac{1}{2}$d. is gained on 3s. what is the gain per cent.?

Ans. L.$20\frac{5}{6}$, or L.20, 16s. 8d.

IV.

How many yards of cloth at 18s. per yard, may be given for 60lbs. of tea at 7s. per lb.?

Ans. $23\frac{1}{3}$ yards.

V.

If the longest end of the beam of a balance be 24 inches, and the shortest 18 inches, what weight suspended at the extremity of the longest arm, will be an equipoise to 60lbs. on the shortest?

Ans. 45lbs.

VI.

A merchantman distant 22 miles, going at the rate of 4½ knots an hour, is pursued by a privateer, whose hourly rate of sailing is 6 knots; after two hours chase the breeze freshened, and the merchantman's rate was encreased to 5.8 knots, and the privateer's to 8.4. In what time will the privateer come up with the merchantman?

Ans. In 9h. 18′ from the commencement of the chase, or in 7h. 18′ after the breeze freshened.

PROBLEM IV.

To find the Square Root of any given number, by the Sliding Rule.

If any number be multiplied by itself, the product, thence arising, is called the square of the given number; and the number itself is called the root. To extract the square root of any given number, is to find that number, which when multiplied by itself will produce the given number.

RULE.

Set the middle division on the line marked C, to 10 on the line D; then, if the given number consists of 2, 4, 6, &c. places of figures, it is to be found on the first, or left hand, half of the line C: but if it consists of 3, 5, 7, &c. figures, it is to be found on the right hand part of the line; opposite to which on the line D, is the square root required.

EXAMPLES.

I.

What is the square root of 64?

The middle division or 1 on the slide, being put to 10 on the line D; then opposite to 64 on the first part of the line C, is 8, the square root on D.

II.

Required the square root of 625?

The slide being placed as before, then opposite 625 on the second part of the slide is 25, the square root on the line D.

PROBLEM V.

To find the Cube Root of any given number.

The cube root of any given number, is that number, by which its square being multiplied, will produce the given number.

RULE.

The cube root of any number, is found by inspection, by those rules which have the single and triple lines of numbers upon one of their edges, as follows.

Find the given number on the triple line of numbers; and, opposite thereto, on the single line, is the root required.—Observing, that if the given number contains 1, 4, 7, 10, &c. figures, it is to be found on the first primary division of the triple line. If it consists of 2, 5, 8, 11, &c. figures, it is to be found on the second or middle principal division. And if it contains 3, 6, 9, 12, &c. figures, it is to be found on the last principal division of the triple line of numbers.

If the given number does not exceed three figures, its root will be below 10; if it is between three, and six figures, the root, will be between 10 and 100, &c.

EXAMPLES.

I.

Required the cube root of 64?

Because 64 contains two places of figures, it is to be found on the middle principal division of the triple line of numbers, opposite to which is 4, the required root, on the single line of numbers.

II.

Required the cube root of 512?

The number 512, consisting of three figures, is to be found on the third principal division, opposite to which is the root 8.

III.

Required the cube root of 4096?

The given number contains four figures, it must, therefore, be found on the first principal division, opposite to which is the root 16.

PROBLEM VI.

Given the diameter of a circle, to find its circumference by the Sliding Rule.

RULE.

Set 113 on B, to 355* on A; then opposite to the given diameter on B, is the circumference on A.

EXAMPLES.

I.

Let the diameter of a circle be 40 inches, required the circumference?

Set 113 on B, to 355 on A; and opposite to 40 on B, is the circumference 125⅔ inches on A.

II.

---

* The proportion of 113 to 355 is very easily remembered. Thus, let the three first odd figures be twice repeated in succession; then, if the three first figures 113 represent the diameter of a circle, the last three, viz. 355, will be the circumference nearly.

II.

The diameter of a circle is 24, what is the circumference?

The slide being placed as before; then opposite to 24 on B, is the circumference 75.3 on A.

PROBLEM VII.

Given the circumference of a circle, to find the diameter.

RULE.

Set 355 on B, to 113 on A; and opposite to the given circumference on B, is the diameter on A.

EXAMPLES.

I.

The circumference of a circle is 264, required the diameter?

Set 355 on B, to 113 on A; then opposite to 264 on B, is the diameter 84 on A.

II.

Let the circumference of a circle be 150 feet, required the diameter?

The slide being placed as before; then opposite to 150 on B, is the diameter 47.8 on A.

PROBLEM VIII.

To find a mean proportional between two given numbers.

RULE.

Set one of the given numbers upon C, to the same upon D; then opposite to the other number on C, is the mean proportional required on D.

EXAMPLES.

I.

Required a mean proportional between 18 and 72?

Set 18 on C, to 18 on D; then opposite to 72 on C, is 36 on D, the mean proportional.

II.

Required a mean proportional between 7 and 24?

Set 7 on C, to 7 on D; then opposite to 24 on C, is 13 nearly on D, the mean proportional.

PROBLEM IX.

Given the radius of a circle, to find the length of an arch of any number of degrees.

RULE.

Set 57.3* on B, to the radius on A; then opposite

---

[*] Or more accurately 57° 17′ 44″.8, the length of an arch equal to the radius.

posite to the given number of degrees on B, is the length of the arch on A.

EXAMPLES.

I.

Let the radius of a circle be 36 inches, required the length of an arch of 65 degrees?

Set 57.3 on B, to 36 on A; and opposite to 65° on B, is the length of the arch, 41 inches, on A.

II.

Required the length of a circular arch of 110 degrees, the radius being 63 inches?

Ans. 121 inches nearly.

PROBLEM X.

Given the transverse and conjugate diameters of an ellipse, to find its periphery.

RULE.

Set 2 on B, to the sum of the diameters on A; and opposite to 3.14 on B, is the periphery nearly on A.

EXAMPLE.

The transverse diameter of an ellipse is 40 inches, and conjugate 30, required its periphery?

Set 2 on B, to 70 the sum of the diameter on A; and opposite to 3.14 on B, is the periphery 110 inches on A.

The

REMARK.

The above rule, generally, gives the periphery less than the truth: and the square root of half the sum of the squares of the diameters, multiplied by 3.14, in general, makes the periphery greater; hence, the mean of these two peripheries, will be near the true length.

SECTION III.

CONTAINING

THE USE OF THE SLIDING RULE,

APPLIED TO

SUPERFICIAL MEASURE.

PROBLEM I.

Given the length and breadth in feet, to find the superficial content in feet.

RULE.

SET 1 on B, to one of the given dimensions on A; then opposite to the other dimension on B, is the superficial content in feet on A.

EXAMPLES.

I.

The length of a room is 23 feet, and breadth 16 feet; required the superficial content of the floor?

Set 1 on B, to 16 on A; and opposite to 23 on B, is 368 feet, the content in A.

II.

How many feet of pavement is opposite to a house whose length is 45 feet, the breadth of the pavement being 7 feet?

Set 1 on B to 7 on A; then opposite to 45 on B is 315 feet on A.

III.

Required the content of a piece of ground, in square yards, whose length is 54 feet, and breadth 35 feet?

Set 9 on B, to 35 on A; and opposite to 54 on B, is the required content 210 yards, on A.

PROBLEM II.

Given the length in feet, and breadth in inches, to find the superficial content in feet.

RULE.

Set 12 on B, to the breadth in inches on A; then opposite to the length in feet on B, is the superficial content in feet on A.

EXAMPLES.

I.

Let the length be 15 feet, breadth 18 inches, required the superficial content?

Set 12 on B, to 18 on A; then against 15 on B, is $22\frac{1}{2}$ feet, the superficial content, on A.

II.

The length of a plank is 26 feet, the breadth at one end 16 inches, and at the other end 22 inches; required the superficial content?

$$\text{The mean breadth is } = \frac{16+22}{2} = 19 \text{ inches.}$$

Now set 12 on B, to 19 on A; then opposite to the length 26 feet on B, is the superficial content $41\frac{1}{6}$ feet on A.

III.

The length of a plank is 31 feet, and mean breadth 17 inches, required the superficial content?

Set 12 on B, to 17 on A; then opposite to 31 on B, is the content 44 feet nearly, on A.

PROBLEM III.

Given the length and breadth in inches, to find the superficial content in feet.

RULE.

Set 144 on B, to the breadth, on A; then opposite to the length on B, is the superficial content in feet on A.

EXAMPLES.

EXAMPLES.

I.

Let the length be 46 inches, and breadth 28 inches, required the superficial content in feet?

Set 144 on B, to 28 on A; then opposite to 46 on B, is 9 feet nearly, the superficial content on A.

II.

A pane of glass is 22 inches by 15 inches, required its content? and supposing the price of glass to be 15d. per foot, the value of the pane is required?

Set 144 on B, to 15 on A; and opposite to 22 on B, is 2 feet $\frac{3}{10}$ths nearly, the contents on A.

Again, set 1 on B, to $2\frac{3}{10}$ths feet on A; and opposite to 15 on B, is $34\frac{1}{2}$ nearly on A: hence the value of the pane of glass is 2s. $10\frac{1}{2}$d. nearly.

III.

The length of a board is 8 feet 7 inches, and breadth 10 inches, required the number of superficial feet it contains?

Set 144 on B, to 10 on A; then against 103 inches, the length, on B, is $7\frac{1}{7}$ feet nearly, the content on A.

IV.

A plank is 18 feet 7 inches long, breadth at one end 11 inches, and at the other end 17 inches, required the number of superficial feet it contains, and its value at 5d. per foot?

Half the sum of 11 and 17 is 14 inches, the mean breadth. Now set 144 on B, to 14, the mean breadth on A; and opposite to the length 223 inches on B, is about $21\frac{6}{10}$ feet, the superficial content on A. Then set 1 on B, to 21.6 on A; and opposite to 5 on B, is 108d. or 9s. on A, the value of the plank.

V.

The length of a room is 16 feet, 4 inches, and breadth 13 feet, 1 inch; required the number of square feet contained in the floor?

The length 16 feet, 4 inches =196 inches; and the breadth 13 feet, 1 inch, =157 inches. Now set 144 on B, to 157 on A; and opposite to 196 on B, is about 213 feet on A, the content required.

PROBLEM IV.

Given the length and breadth in feet, or in feet and inches, to find the superficial content in square yards.

RULE.

Find the superficial content in feet by one of the preceding problems. Then set 9 on B, to 1 on A; and opposite to the content in feet on B, is the content in square yards on A.

EXAMPLES.

I.

Let the length be 27 feet, breadth 20 feet; required the content in square yards?

Set 1 on B, to 20 on A; and against 27 on B, is 540 on A, the content in feet. Now set 9 on B, to 1 on A; and opposite to 540 on B, is 60 on A, the square yards required.

Or, in this case, where the length and breadth are given in feet, set 9 on B, to 20 on A; then opposite to 27 on B, is 60 on A; as before.

II.

The length of a room is 16½ feet, and breadth 14¼ feet; required the content of the floor in square yards?

Set 9 on B, to 14¼ on A; and against 16½ on B, is 26⅛ on A, the content of the floor in square yards. Hence it would require 26⅛ yards of carpeting, of 1 yard in breadth, to cover the floor.

III.

Required the expence of painting one side of a room, whose length is 22½ feet, and height 11 feet, at 5d. per square yard?

Set 9 on B, to 11 on A; and opposite to 22½ on B, is 27½, the square yards, on A. Then set 1 on B, to 5 on A; and against 27½ on B, is 137½d. on A; which divided by 12, gives 11s. 5½d.

IV.

The length of a piece of side pavement is 86 feet, and breadth within the crib-stone 4 feet, 6 inches; required the number of square yards it contains, and the expence of workmanship in laying down the same at 3½d. per square yard?

Set 9 on B, to 4½ on A; and opposite to 86 on B, is 43 on A, the area in square yards. Now set 1 on B, to 3½ on A; and opposite to 43 on B, is 150½ on A; which reduced, gives 12s. 6½d.

V.

Required the number of yards of paper sufficient for one side of a room whose length is 22½ feet and 10 feet high, the breadth of the paper being 21 inches?

Set 9 on B, to 22½ on A; and opposite to 10 on B, is 25 on A, the number of square yards in the side of the room. Then, as the paper is only 21 inches in breadth, set 21 on B, to 36 on A; and opposite to 25 on B, is 43 nearly on A, the yards of paper required.

PROBLEM V.

Given the base, and perpendicular of a plane Triangle, to find its Area.

RULE.

Set 1 on B, to half the base, or perpendicular on A; then opposite the other term on B, is the area of the triangle on A.

EXAMPLES.

I.

Let the base of a triangle be 24 feet, and the perpendicular from the vertical angle to the base, 15 feet; required its area?

Set 1 on B, to 12 on A; and opposite to 15 on B, is the area 180, on A.

II.

Required the area of a triangle, in square yards, whose base is 63 feet, and perpendicular altitude 42 feet?

Set 9 on B, to 63 on A; and opposite to 21, half the perpendicular, on B, is 147, the area in square yards, on A.

PROBLEM VI.

To find the several numbers for comparing the diameter, circumference, and area of a circle with each other; and with its equal, inscribed, and circumscribed squares.

RULE.

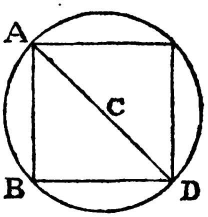
If the diameter of a circle is 1, its circum-

ference

ference is 3.14159 26535 89793, &c.; or, for common practice, 3.1416. Then, since a circle may be conceived to be a regular polygon, composed of an infinite number of triangles, and the area of each, being equal to half the base multiplied by the perpendicular; the area of the circle will, therefore, be equal to half the circumference, multiplied by half the diameter, or the radius; that is, equal to $\frac{3.1416}{2} \times \frac{1}{2} = .7854$.

The side of a square, equal in area to a circle, whose diameter is 1, is equal to the square root of .7854, that is .8862.

Let A B, and B D, be sides of a square, inscribed in a circle, whose center is C; then in the right angled triangle A B D, the square of A D, is equal to the sum of the squares of A B and B D; but, as these sides are equal, the square of A D, is double the square of A B; and, consequently, the square root of half the square of A D, is equal to A B. Now, since A D is 1, its square root is also 1, the half is .5, and the square root of .5 is .7071; or, to A D as a radius, then A B is the sine of 45 degrees.



The side of the circumscribed square is equal to 1, the diameter of the circle; and its area is also 1. Hence, the area of the circumscribed square, is double that of the inscribed square.

Let the circumference of a circle be 1, then its diameter is equal to 1, divided by .3,1416= .3183; and its area is equal to the square of .3183 multiplied by .7854=.07958. The side of an equal square is equal to .2821, the square root of .07958. The side of the inscribed square is equal to the diameter, multiplied by the sine of 45 degrees, =.3183×.7071,=.225079; or it is equal to .7071 divided by 3.1416. And the area is equal to .05066, the square of .225079. The side of the circumscribed square being equal to the diameter .3183; its area is, therefore, .10132, the square of .3183.

If the area of a circle be 1, its diameter will be 1.128, which is the square root of 1, divided by .7854. The circumference will be equal to the diameter multiplied by 3.1416, =1.128×3.1416 =3.545 nearly; and if the area is 12, the circumference will be =3.545×12= 42.54. The area of a circle being 1, the side of an equal square is, evidently, 1. The side of the inscribed square is equal to the diameter multiplied by the sine of 45 degrees =1.128×.7071=.7979:

And, the area of the inscribed square is equal to .6366, the square of .7979.

The side of the circumscribed square is equal to the diameter, $=1.128$; and its area is equal to 1.2732, the square of 1.128. Or, since the area of the circumscribed square, is double that of the inscribed square, it is, therefore, equal to $.6366 \times 2 = 1.2732$.

PROBLEM VII.

Given the diameter of a circle, to find its area.

RULE.

Set .7854 on C, to 10 on D; and opposite to the diameter on D, is the area required, on C, in the same terms as the diameter.

But if the diameter be given in inches, and the area required in feet; then set .7854 on C, to 12 on D; and opposite to the diameter on D, expressed in inches, is the area in square feet on C.

EXAMPLES.

I.

Required the area of a circle in square inches, whose diameter is 6 inches?

Set .7854 on the slide, to 10 on D; then opposite to 6 on D, is 28.3, nearly on C, the area in square inches.

II.

II.

Let the diameter of a circle be 30 inches, required its superficial content in feet?

Set .7854 on C, to 12 on D; and against 30 on D, is 4 feet $\frac{9}{10}$ths nearly, the area on C.

REMARK.

Upon the girt line there are two points marked w g, and a g, signifying *wine*, and *ale gallons*. By these, which are called *gauge points*, the area of a circle in wine, or ale gallons, is easily found, as follows.

Set 1 on C, to the gauge point on D; and opposite to the diameter of the circle on D, is the area on C.

EXAMPLE.

Let the diameter of a circle be 34 inches, required its area in wine and ale gallons?

Set 1 on C, to w g on D; then opposite to the diameter 34 on D, is 4, the area in wine gallons on C.

Again, set 1 on C, to a g, on D; and opposite to 34 on D, is 3.2, the area in ale gallons, on C.

PROBLEM VIII.

Given the circumference of a circle, to find its area.

RULE.

Set 1 on C, to 3.545* on D; then opposite to the

---

* The number 3.545 is the circumference of a circle whose area is 1.

the given circumference on D, is the superficial content required on C, in the same terms as the circumference.

But if the circumference be given in inches, and the area required in feet; then set 1 on C to 42.54[*] on D; and opposite to the circumference in inches on D, is the area in feet on C.

I.

Let the circumference of a circle be 44 inches, required its area in square inches?

Set 1 on C, to 3.545 on D; and opposite to 44 the circumference on D, is 154 on C, the area in square inches.

II.

The circumference of a circle is 168 inches, required the area in square feet?

Set 1 on C, to 42.54 on D; and opposite to 168 on D, is 15 feet $\frac{6}{10}$ths on C, the area required.

PROBLEM IX.

Given the area of a circle to find its diameter, and circumference.

RULE.

To find the diameter.

Set .7854 on C, to 10 on D; and opposite to the area on C, is the diameter on D.

---

[*] The number 42.54 is the circumference of a circle whose area is 12.—See Prob. VI.

To find the circumference.

Set 1 on C, to 3.545 on D; and opposite to the area on C, is the circumference on D.

EXAMPLE.

Required the diameter and circumference of a circle, whose area is 25 inches?

Set .7854 on C, to 10 on D; and opposite to 25 on C, is the diameter 5.64 on D.

Again, set 1 on C, to 3.545 on D; and opposite to 25 on C, is the circumference 17.7 on D.

PROBLEM X.

Given the diameter of a circle, to find the side of a square, whose area shall be equal to that of the circle.

RULE.

Set 1 on B, to .886 on A; and opposite to the given diameter on B, is the side of a square of equal area on A.

EXAMPLE.

Required the side of a square, equal in area to a circle, whose diameter is 34 inches?

Set 1 on B, to .886 on A; then opposite to to 34 on B, is 30 nearly, the side of the equal square on A.

PROBLEM XI.

Given the circumference of a circle, to find the side of a square, whose area shall be equal to that of the given circle.

RULE.

Set 1 on B, to .282 on A; then opposite to the given circumference on B, is the side of a square on A, equal in area to the given circle.

EXAMPLE.

Required the side of a square, equal in area to a circle whose circumference is 40 inches?

Set 1 on B, to .282 on A, and opposite to 40 on B, is 11.2 on A, the side of a square whose area is equal to that of the given circle.

PROBLEM XII.

Given the diameter of a circle, to find the side of its inscribed square.

RULE.

Set 1 on B, to .707 on A; and opposite to the diameter on B, is the side of the square required on A.

EXAMPLE.

Let the diameter of a circle be 20 inches, required the side of its greatest inscribed square?

Set 1 on B, to .707 on A; then opposite to 30 on B, is 21 inches on A, the side of the square required.

PROBLEM XIII.

Given the circumference of a circle, to find the side of its inscribed square.

RULE.

Set 1 on B, to .225 on A; then opposite to the given circumference on B, is the side of the inscribed square on A.

EXAMPLE.

Let the circumference of a circle be 40 inches, required the side of its greatest inscribed square?

Set 1 on B, to 225 on A; then opposite to 40 on B, is 9 inches, on A, the side of the greatest inscribed square.

PROBLEM XIV.

Given the transverse and conjugate diameters of an ellipse, to find its area.

RULE.

Find a mean proportional between the two diameters, by Prob. VIII. page 24. Then set 1 on D, to .7854 on C, and opposite to the mean proportional on D, is the required content on C.

EXAMPLES.

I.

Required the area of an ellipse, whose transverse diameter is 36 inches, and conjugate diameter 16 inches?

Set 36 on C, to 36 on D; and opposite to 16 on C, is the mean proportional 24 on D. Now set 1 on D, to .7854 on C; and opposite to 24 on D, is 452, the area on C.

II.

Let the transverse diameter of an ellipse be 49 inches, and the conjugate 25 inches, required its area? Answer 962 square inches.

PROBLEM XV.

Given the circumference of the base, and the slant side of a right cone, to find its superficial content.

RULE.

Set 1 on B, to half the circumference on A; and opposite to the slant side on B, is the superficial content on A.

EXAMPLE.

The circumference of the base of a right cone is 24 inches, and the slant side 30 inches, required its superficial content?

Set 1 on B, to 12 on A; and against 30 on B, is the superficial content, 360 inches on A.

REMARKS.

The superficial content of a right pyramid, is found in the same manner as that of the cone.

The surface of the frustum of a cone, or pyramid,

mid, is found by compleating the cone, or pyramid: which may be done by setting the bottom diameter on B, to the top diameter on A; then against the slant side of the frustum on B, is a number on A, which added to the slant side of the frustum, compleats the cone. Now, the surface of the whole cone, and the additional part, being found, by this problem, their difference will be the surface of the frustum.

The surface of a cylinder is found by multiplying its circumference by its length.

PROBLEM XVI.

Given the diameter of a sphere, to find its superficial content.

RULE.

With the given diameter find the circumference by Prob. VI. page 22; then set 1 on B, to the diameter on A; and opposite to the circumference on B, is the superficial content on A.

Or, with the given diameter, find the area of the circle, by Prob. VII. page 36, and four times this area will be the surface.

EXAMPLE.

Let the diameter of a sphere be 8 inches, required its superficial content?

Set 113 on B, to 355 on A; and opposite to 8 on B, is the circumference 25.13 on A. Then set 1 on B, to 8 on A; and opposite to 25.13 on B, is the superficial content 201 inches on A.

Or, set 7854 on C, to 10 on D; and opposite 8 on D is the area 50.26 on C; four times which, or 201, is the superficial content.

PROBLEM XVII.

To find the superficial content of a spheroid.

RULE.

Set .318 on B, to the conjugate axis on A; then opposite to the transverse axis on B, is the superficial content on A.

EXAMPLE.

The transverse diameter of a spheroid is 9 inches, and the conjugate 5 inches, required its superficial content?

Set .318 on B, to 5 on A; then opposite to 9 on B, is 141, the superficial content on A.

PROBLEM XVIII.

Given the diameter of the base, and perpendicular height of the segment of a sphere, to find its superficial content.

RULE.

Find the sum of the squares of height, and half the base; then, set 1 on B, to 3.14 on A;

and opposite to the above sum on B, is the superficial content on A.

EXAMPLE.

Let the base be 30 inches, and height 9 inches, of the segment of a sphere, required its superficial content?

Half of 30 is 15, and its square is		225
Square of 9	=	81
Sum	....	306

Now, set 1 on B, to 3.14 on A; and opposite to 306 on B, is the superficial content, 961 inches on A.

REMARK.

This may also be found as follows:

Divide half the diameter of the base of the segment, by its height, and to the quotient add the height, the sum will be the diameter of the sphere. Then the product of 3.1416 by the diameter of the sphere, and height of the segment, will be the superficial content of the segment.

Or, with the diameter of the sphere, find its superficial content by Prob. XVI. page 43. Then set the diameter of the sphere on B, to the height of the segment on A; and opposite the superficial content of the sphere on B, is that of the segment on A.

Hence, the content of any circular zone may be found, by computing the surface of each extreme segment, reckoned from the same pole, and their difference will be that required.

PROBLEM XIX.

Given the side of any regular polygon, from three to twelve sides inclusive, to find its area.

RULE.

Set the number, from the following table, answering to the given polygon, on C, to 10 on D; and opposite to the given side of the polygon on D, is its superficial content on C.

AREAS OF REGULAR POLYGONS,
THE SIDE BEING 1.

NAMES.	N° of Sides.	AREA.
Equilateral Triangle··	has 3 Sides	0.4330127
Square ············	4	1.0000000
Pentagon ··········	5	1.7204774
Hexagon ··········	6	2.5980762
Heptagon ··········	7	3.6339124
Octagon············	8	4.8284271
Nonagon ···· ····	9	6.1818242
Decagon············	10	7.6942088
Undecagon··········	11	9.3656399
Dodecagon··········	12	11.1961524

EXAMPLE

EXAMPLE.

Required the area of a pentagon, whose side is 13 inches?

Set 1.72 on C, to 10 on D; and opposite to 13 on D, is the superficial content 290 on C.

REMARK.

The numbers in the preceding table may be found as follows.

Find the angle at the center, by dividing 360° by the number of sides; then a perpendicular drawn from the center, to one of the sides will be equal to half the base multiplied by the natural co-tangent of half the angle at the center; that is $=\frac{1}{2}\times$ co-tang. half angle at center; and the area of the triangle is equal to half the base multiplied by the perpendicular, $=\frac{1}{2}\times\frac{1}{2}\times$ co-tang. of half the angle at the center $=\frac{1}{4}\times$ co-tang. half angle: which multiplied by the number of sides, the product will be the superficial content of the polygon.

Thus, to find the tabular number for a pentagon. The angle at the centre is $=\frac{360°}{5}=72°$, and its half 36°, and since the figure contains five sides, the expression will be $=\frac{5}{4}\times$ co-tangent $36°=1\frac{1}{4}\times 1.37638\ 19205=1.7204774006$.

SECTION IV.

CONTAINING

THE USE OF THE SLIDING RULE,

APPLIED TO

SOLID MEASURE.

PROBLEM I.

Given the length in feet, and the breadth or thickness in inches, of equal sided timber, to find the solid content in feet.

RULE.

Set 12 upon D, to the length on C; then opposite to the breadth or thickness on D, is the solid content in feet on C.

EXAMPLES.

I.

The length of a log is 20 feet, and breadth 16 inches, the thickness being the same, required the number of cubic feet it contains?

Set 12 on D, to 20 on C; then opposite to 16 on D, is $35\frac{1}{2}$ feet, the content on C.

II.

Let the length of a log be 28 feet, and the side of the square $14\frac{1}{2}$ inches, required the solid content?

Set 12 on D, to 28 on C; and opposite to $14\frac{1}{2}$ on D, is $40\frac{7}{8}$ on C, the content in cubic feet.

III.

A block of marble is 7 feet long, and each side 2 feet 4 inches; required the number of solid feet it contains?

Set 12 on D, to 7 on C; then opposite to 28 inches on D, is 38 feet on C, the solid feet required.

PROBLEM

PROBLEM II.

Given the length in feet, the breadth and thickness in inches, to find the solid content in feet.

RULE.

Find a mean geometrical proportional between the breadth and thickness by Prob. VIII. page 24; then set 12 on D, to the length on C; and opposite to the mean proportional on D, is the cubic feet required on C.

EXAMPLES.

I.

The length of a log is 22 feet, breadth 19 inches, and thickness 13½ inches; required the number of cubic feet it contains?

Set 19 on C, to 19 on D; and opposite to 13½ on C, is 16 on D, the mean proportional. Now set 12 on D, to 22 on C; and opposite to 16 on D, is 39 on C, the required content in solid feet.

II.

Let the length of a log of mahogany be 13½ feet, breadth 2 feet 11 inches, and thickness 1 foot, 8 inches; required the content?

Set 20 on C, to 20 on D; and opposite to 35 on C, is 26½ on D, the mean proportional. Now set 12 on D, to 13½ on C; and opposite to 26½ on D, is 65½ feet on C, the required content in solid feet.

III.

A bale is $8\frac{1}{4}$ feet long, $3\frac{1}{2}$ feet broad, and 2 feet 2 inches deep; how many cubic feet doth it contain?

Set 26 on C, to 26 on D; and opposite to 42 on C, is 33 inches on D, the mean proportional. Now set 12 on D, to $8\frac{1}{4}$ on C; and opposite to 33 on D, is 62 feet on C, the answer. And, since five cubic feet make a barrel bulk, therefore, set 5 on B, to 1 on A; and opposite to 62 on B, is $12\frac{2}{5}$ barrels bulk on A.

IV.

Required the number of feet of plank $3\frac{1}{2}$ inches thick, to make a load, or 50 cubic feet?

Set $3\frac{1}{2}$ on B, to 12 on A; and opposite to 50 on B, is 240 on A.

PROBLEM III.

Given the length in feet, the breadth and thickness at each end in inches, to find the solid content in feet.

RULE.

Find the mean proportional between the breadth and thickness at each end, and let half their sum be called the side of the mean square. Then set 12 on D, to the length on C; and opposite to the side of the mean square on D, is the solidity required on C.

EXAMPLES.

EXAMPLES.

I.

The length of a log is 32 feet, breadth at one end 23 inches, and thickness 14 inches; and the breadth at the other end 19 inches, and thickness 12¾ inches; required the content?

Set 14 on C, to 14 on D; and opposite to 23 inches on C, is 18 on D. Again, place 19 on C, to 19 on D; and opposite to 12¾ on C, is 15½ on D. Now half the sum of 18 and 15½ is 16¾ inches, the side of the mean square.

Then set 12 on D, to 32 on C; and opposite to 16¾ on D, is 62 on C, the cubic feet required?

II.

Let the length be 27½ feet, breadth at the greater end 20 inches, and thickness 12 inches; breadth at the less end 17 inches, and thickness 10 inches; required its content?

By proceeding as before, the mean proportional of the greater end will be 15½ inches, and of the less 13; half the sum is 14¼ inches. Now set 12 on D, to 27½ on C; and opposite to 14¼ on D, is $38\frac{7}{9}$ on C, the number of cubic feet required?

PROBLEM IV.

Given the length of a tree in feet, and mean diameter in inches, to find its content.

RULE.

Set 1 upon C, to the mean diameter on D; and opposite to .7854 on C, is the side of a square equal in area to the circle whose diameter is given.

Now set 12 on D, to the length on C; and opposite to the side of the square on D, is the solid content on C.

REMARK.

In round timber, which tapers gradually, the circumference may be taken at the middle for the mean girt; or half the sum of the circumferences at each end. If it tapers irregularly, the circumference is to be taken at the ends and at equal intervals; and their sums, divided by the number of times the circumference has been taken, will be the mean girt or circumference. Thus let the circumference at the ends, and at equal intervals, be 35, 28, 23, 20, and 14. Now their sum 120, divided by their number 5, gives 24 inches, the mean girt.

EXAMPLES.

I.

The length of a tree is 42 feet, and mean diameter 1 foot, 8 inches, required its solid content.

Set 1 on C, to 20 on D; and opposite to .7854 on C, is 17¾ inches on D, the side of the square. Now set 12 on D, to 42 on C; and opposite to 17¾ inches on D, is 91½ feet on C, the solidity.

II.

Let the length of a spar be 52 feet, and mean diameter 14 inches, required the solid content.

Set 1 on C, to 14 on D; and opposite to .7854 on C, is about 12.4 on D. Now set 12 on D, to 52 on C; and opposite to 12.4 on D, is about 55.6 feet on C, the solidity required.

PROBLEM V.

Given the length of a tree in feet, and its circumference in feet or inches, to find its solid content in feet.

RULES.

I.

The circumference being given in feet.

Set 3.545 on D, to the length on C; and opposite to the circumference on D, is the solid content on C.

II.

II.

The circumference being given in inches.[*]

Set 42.54 on D, to the length in feet on C, then against the circumference on D, is the solid content on C.[†]

The circumference being given in feet.

EXAMPLES.

I.

The length of a tree is 46 feet, and circumference 3 feet, required the solid content?

Set

---

[*] This computation is also very easily performed by Logarithms, as follows:

RULE.

To the constant logarithm 6.742429, add the logarithm of the length, and twice the logarithm of the circumference, the sum, rejecting 10 from the index, will be the logarithm of the solid content in feet.

EXAMPLE.

Let the length of a spar be 35 feet, and the circumference 30 inches; required its solid content.

Constant logarithm	6.742429
Length 35 feet··log................	1.544068
Circumference 30, log. 1.477121×2=	2.954242
Solidity···· 17. 4 feet	1.240739

[†] Or multiply the circumference in inches by .2821 (which is the side of a square equal to a circle whose circumference is 1); then multiply the square of this product by the length in feet, which being divided by 144, the quotient will be the answer in cubic feet.

Set 46 on C to 3.545 on D; then opposite to the circumference 3 feet on D, is very nearly 33 feet, the content on C.

II.

The length of a tree is 50 feet, and mean circumference 4½ feet, required the content?

The length 50 feet on C, being put to 3.545 on D, the beginning of the line C is removed to the right hand of 4½ on D. In this case, therefore, take any multiple or part of the length, as its double, which is 100, and place opposite to 3.545 on D, and against 4.5 on D is 161 on C, the half of which 81½ is the number of cubic feet required.

The circumference being given in inches,

EXAMPLES.

I.

Let the length of a tree be 20 feet, and mean circumference 29 inches, required its solid content?

Set 20 on C, to 42.54 on D; then opposite to 29 on D, is $9\frac{3}{10}$ feet, the solid content on C.

II.

The length of a tree is 48 feet, and mean circumference 35 inches, required the solid content?

The length on C being set to 42.54 on D, there is no part of the slide opposite to the circumference on D. In this case, therefore, take any aliquot part of the length, as a fourth, which is 12. Now set 12 on C, to 42.54 on D; and opposite to 35 on D, is 8⅛ on C; which, multiplied by 4, gives 32½, the solid feet required.

III.

The length is 56 feet, and circumference 40 inches, required the solid content?

Set 56 on C, to 42.54 on D; and opposite to 40 at the left hand end of the line D, is 49½ feet, the answer on C.

IV.

Let the length of a tree be 45 feet, and circumference 32 inches, required the content?

The length on C being set to 42.54 on D, there is no part of the slide opposite to 32 on D. Let therefore one-third of the length, which is 15, on C, be set to 42.54 on D; and opposite to 32 on D, is 8.5 on C; which multiplied by 3, gives 25.5, the solid content.

Instead of taking aliquot parts of the length, the content may be found more generally by the folowing rule:

Set 10 and the end of the slide on C, to the circumference on D, and observe the number on C, which is opposite to 42.54 on D. Now set this number on B, to the middle Division on A; and opposite to the length on B, is the content on A.

Thus, in the above example, set 10 at the end of the line C, to 32 on D; and opposite to 42.54 on D, is 17.6 on C. Now set 17.6 on B, to 10 on A; and opposite to the length 45 feet on B, is 25.5, the content on A.

Again, in example second, set 10 at the end of the slide on C, to the circumference 35 on D; and opposite to 42.54 on D, is 14.7 on C. Now set 14.7 on B, to the middle 10 on A; and opposite to the length 48 on B, is 32½ feet, the solid content on A.

REMARK.

The number of cubic feet in a box or bale being found by the preceding rules, its barrel bulk is obtained by doubling the feet, and pointing off the right hand figure as a decimal.

Thus, in example third, page 50, the contents of the bale in cubic feet is 62, which being doubled, and the right hand figure pointed off, gives 12.4, the barrel bulk required.

PROBLEM

PROBLEM VI.

Given the length, and the breadth, and depth in feet, to find the solid content in yards.

RULE.

I.

The section of the solid being rectangular.

Find a mean proportional between the breadth and thickness by Problem VIII. page 24. Then set the length in feet on the line C, on the slide, to 5.2* on the line D, and opposite to the mean proportional on D, is the content in cubic yards on C.

II.

The section of the solid being triangular.

Find a mean proportional between the perpendicular and half the base, and proceed as before.

EXAMPLES.

I.

Required the number of cubic yards in a portion of a canal, whose length is 150 feet, mean breadth 19 feet, and depth 8½ feet?

Set 8½ on C, to 8½ on D; and opposite to 19 on C, is 12¾ on D, the mean proportional.

Now set the length 150 feet on C, to 5.2 on D;

---

[*] Or rather 5.196, which is the square root of 27, the number of cubic feet in a cubic yard.

D; and opposite to 12¾ on D, is the content in cubic yards 897, on C.

II.

The length of a piece of through-cutting* is 30 feet, mean breadth 43 feet, and mean depth 6⅔ feet, required the content in cubic yards?

Set 6⅔ on C, to 6⅔ on D; and opposite to 43 on C, is about 16.9 on D. Now set 30 on C, to 5.2 on D; and opposite to 16.9 on D, is about 319 yards, the solid content on C.

III.

Let the length of a piece of side-cutting be 80 feet, base 26 feet, and perpendicular height 14 feet, required the content in cubic yards?

In this example the section is a triangle, and therefore the mean proportional between the base and half the perpendicular is to be found, which, by proceeding as before, will be 13½ nearly. Now set 80 on C, to 5.2 on D, and opposite to 13½ on D, is 539, the content in cubic yards on C.

REMARK.

---

* *Through-cutting*, *side-cutting*, *side-banking*, &c. are terms used in road-making. The first of these terms is when a piece of rising ground is cut through in order to make the road more upon a level. When a road is made along the side of a hill, the quantity cut out of the declivity is called *side-cutting;* and *side-banking* is the quantity to be laid down to make up a part of the side of the road, that it may be of the proper breadth and heighth.

REMARK.

In order to render the calculations for ascertaining the number of cubic yards in a piece of side-cutting as simple as possible, let the measures be taken in the following manner:

Stretch a line from the upper part of the side-cutting to the nearest part of the outer edge of the road, and let the length of this line be called the *base;* then, by means of a divided rod, measure the nearest distance between this line and the intersection of the horizontal and sloping planes, which call the *perpendicular.* Now multiply the length of the piece of side-cutting by the perpendicular, and by half the base, each being expressed in feet; and this product, divided by 27, will give the solid content in cubic yards.

When the horizontal and slant sides of the side-cutting are considerable, and nearly at right angles to each other, the perpendicular will be so long as not to be easily measured by the above method. In this case, therefore, the three sides are to be measured. Now from half the sum of the three sides, subtract each side separately; then extract the square root of the product of half the sum of the sides by the three differences; multiply this root by the length of the section, and the product, divided by 27, will be the content in cubic yards.

Or with the three sides calculate the angle at the horizontal extremity. With this angle, the distance between the opposite edges of the side-cutting, the base or horizontal measure, and the length of the section, the content in cubic yards may be found.

The limits assigned to this tract will not admit of entering into a detail of the most proper methods of taking the various measures in side-cutting, side-banking, &c. nor into the different methods of making the calculations from these measures.

PROBLEM VII.

Given the diameter, or circumference, and the length of a cylinder, to find its solid content.

RULES.

I.

The diameter being given.

Set the length of the cylinder on C, to 1.13* on D; then opposite to the diameter on D, is the content on C, in the same measure.

If the length is given in feet, and the diameter in inches.

---

* Or more accurately 1.12838, the diameter of a circle whose area is 1; or the square root of the reciprocal of .7854.

Set the length in feet on C, to 13.54* on D; and opposite to the diameter in inches on D, is the content in feet on C.

II.

The circumference being given,

Set the length on C to 3.54 on D; and opposite to the circumference on D, is the content on C, in the same measure.

If the length is given in feet, and the circumference in inches,

Set the length in feet, on C, to 42.54 on D; and opposite to the circumference in inches on D, is the content in feet, on C.

EXAMPLES.

I.

Let the length of a cylinder be 60 inches, and its diameter 24 inches, required its content in inches, and also in feet?

Set 60 on C, to 1.13 on D; and opposite to 24 on D, is 27143 on C, the content: or, set 5 feet the length on C, to 13.54 on D; and opposite 24 inches on D, is 15.7 feet, the solid content on C.

---

* The diameter of a circle whose area is 144, and, therefore, equal to 1.12838 multiplied by 12, the square root of 144.

II.

The circumference of a cylinder is 30 inches, and its length 4 feet, required its solid content in inches, and also in feet?

Set the length 48 inches on C, to 3.54 on D; and opposite to 30 on D, is the content, 3438 cubic inches, on C.

Again, set 4 feet, the length, on C, to 42.54 on D; and as the slide is now placed, the right hand extremity of the line C, is considerably to the left of 30 on D; observe, therefore, if any division on the second part of the line C, coincides with a division on the line D; in this example, 50 on the second part of the line C, coincides with 15 on D; therefore, draw out the slide until 50 on the first part of C, coincides with 15 on D, then opposite to the circumference 30 on D, is 1.99 the content, in cubic feet, on C.

Or set 4 feet on C, to 3.54 on D, and opposite the circumference 2.5 on D, is 1.99 the content on C.

PROBLEM VIII.

Given the diameter, or circumference, of the base of a cone, and its perpendicular height, to find its solid content.

RULE.

Proceed as directed in the last problem, with this difference only, that one-third of the perpendicular height is to be used instead of the length of the cone.

EXAMPLES.

I.

The diameter of the base of a cone is 25 inches, and perpendicular height 7 feet; required its solid content in inches?

Set 28 inches, one-third of the perpendicular height, on C, to 1.13 on D; and opposite to the diameter, 25 inches, on D, is the solid content, 1412 inches, on C.

II.

The height of a cone is 12 feet, and the circumference of the base 54 inches; required its solid content in feet;

Set 4 feet, one-third of the altitude, on C, to 42.54 on D; then opposite to 54 inches, the circumference on D, is 6.446 feet on C, the solid content.

REMARK.

If the base of a cone is elliptical, find a mean proportional between the transverse and conjugate diameters, with which, and the height, proceed as before.

PROBLEM

PROBLEM IX.

Given the side of the base, and perpendicular altitude of a pyramid, to find its solid content.

RULE.

Find the area of the base, by prob. I. chap. II. Then, set 1 on B, to the area of the base on A; and opposite to one-third of the perpendicular height on B, is the solid content on A.

EXAMPLE.

The side of the base of a square pyramid is 36 inches, and its perpendicular height is 63 inches; required its solid content in feet?

Set 1 on C, to 10 on D; and opposite to 36 on D, is 1300 nearly, the area of the base on C. Then, set 1728 on B, to 1300 on A; and opposite to 21 inches, one-third of the perpendicular height, on B, is the solidity of the pyramid, 15¾ cubic feet, on A.

REMARK.

If the base be triangular, pentagonal, &c. find the area of the base, and proceed as before.

PROBLEM X.

Given the diameter or circumference of a globe, to find its solid content.

RULES.

I.

The diameter being given.

Set the diameter on C, to 1.38 on D; and opposite to the diameter on D, is the content on C, in the same measure.

If the diameter is given in inches, and the content required in feet.

Set the diameter in inches, on C, to 57.4 on D; and opposite to the diameter in inches, on D, is the content, in feet, on C.

II.

The circumference being given.

Set the circumference on C, to 7.7[*] on D; and opposite to the circumference on D, is the content on C, in the same measure as the circumference was expressed.

If the circumference is given in inches, and the content required in feet.

Set the circumference in inches on C, to 320[†] on D; and opposite to the circumference on D, is the content in feet on C.

EXAMPLES.

I.

The diameter of a sphere is 2 feet, 6 inches; required its solid content in feet, and also in inches?

---

[*] Or 7.6923, the square root of the reciprocal of .0169, the solid content of a sphere, whose circumference is 1.

[†] Or 319.76, the square root of the quotient arising from the division of 1728, by .0169.

Set the given diameter, 30 inches, on C, to 1.38 on D; and opposite to 30 on D, is the content in inches, 14137, on C.

Again, set 30 on C, to 57.4 on D; and opposite to 30 on D, is 8.18, the content in feet, on C.

Or, set 2 feet, 5 tenths on C, to 1.38 on D; and opposite to 2.5 on D, is 8. 18 on C, as before.

PROBLEM XI.

To find the solid content of the sector of a sphere.

RULE.

Find the superficial content of the segment by Prob. XVIII. page 44. Then set 1 on B, to one-third of the radius of the sphere on A; and opposite to the surface of the segment on B, is the solid content of the sector on A. Or multiply the spherical surface of the sector by one-third of the radius of the sphere, the product will be the solid content.

EXAMPLE.

The diameter of a sphere is 34 inches, and the diameter of the segment 30 inches, required the solidity of the corresponding sector?

By Prob. XVIII. the surface of the segment is 961.33, which multiplied by $5\frac{2}{3}$, one-third of 17 the radius of the sphere, the product 5447.54 is the solid content of the sector.

PROBLEM XII.

Given the diameter and height of the segment of a sphere, to find its solid content.

RULE.

With the given terms, find the solid content of the sector of the sphere, by last Problem. And with the diameter of the segment, and its distance from the centre of the sphere, find the solid content of the cone, by Prob. VIII. page 63: then, the difference between these, will be the solid content of the segment.

Or, to three times the square of the base, add four times the square of the height, multiply the sum by one-sixth of the height, and by .7854, and the product will be the solid content.

EXAMPLE.

Let the base of the segment of a sphere be 30 inches, and height 9 inches; required its solid content?

By the preceding Problem, the content of the sector is . . . . =5447.54.

And the content of the cone whose base is 30, and height 9 inches, is	1884.96
Solid content of the segment	3562.58.

By the second Rule.

Base	30	Height	9
	30		9
Square ..	900	Square	81
Multiply by	3	Multiply by ..	4
Product ..	2700	Product	324
			2700

Sum 3024

One-sixth height $=\frac{9}{6}=$ .... $1\frac{1}{2}$

3024

1512

The product is .... =4536; which being mutiplied by .7854, the product is 3562.57, the solid content, as before.

PROBLEM XIII.

To find the solid content of the frustum of a pyramid, or of a cone; the sides of the squares at the top and bottom of the frustum of a pyramid, or the diameters of the circular ends of the frustum of the cone, and the perpendicular height, being given.

RULE.

By the sliding rule, or otherwise, find the squares of the top and bottom sides, and also the product of these sides: the product of the sum of these three numbers, by one-third of the height, will be the solid content of the frustum of a pyramid in inches, if all the terms are given in inches; which may be reduced to feet by dividing the above product by 1728.—Or, by setting 1728 on B, to the product on A; and opposite to 1 on B, is the solid content of the frustum of the pyramid, in feet, on A. If the sides of the squares are given in inches, and the height in feet, the divisor is 144.

If the solid is the frustum of a cone, find the solid content of a pyramid, whose sides are the diameters of the top and bottom circles, and the altitude the same as the cone; then, set 1 on B, to .7854 on A; and opposite to the content of the pyramid, on B, is the solid content of the frustum of the cone on A.

EXAMPLES.

I.

In a square pyramid, let the side of the square at the bottom be 42 inches, and at the top 31 inches, and the perpendicular height 27 inches; required the content in cubic feet?

Square of bottom side····$=42\times42=1764$
———of top side ······$=31\times31=961$
Product of top and bottom$=42\times31=1302$

Sum ············ ··········· · 4027
One-third height ··········$=\frac{27}{3}=$ 9

Solid content in inches············36243

Now set 1728 on B, to 36243 on A; and opposite to 1 on B, is the content 21 feet nearly, on A.—Or, set 1728 on B, to 4027 on A, and opposite to 9 on B, is 21, the content on A.

II.

Let the diameter of the base of the frustum of a cone be 50 inches, and that of the top 34 inches; and the perpendicular height 66 inches: Required its solid content in feet?

Square bottom diam.=50×50=2500
Square top diam ···=34×34=1156
Product of diam.····=50×34=1700

Sum························5356

Now, set 1728 on B, to 5356 on A, and opposite to 33, one-third of the height, on B, is 102.3 the content of the frustum of a pyramid on A.—Then, set 1 on B, to .7854 on A; and opposite to 102.3 on B, is the solid content of the frustum of the cone 80, on A.

PROBLEM XIII.

To find the content of a solid, bounded by six planes, of which the bases are parallel, but not similar to each other.[*]

RULE.[†]

Find the product of the breadth and thickness

---

[*] If the bases are polygons, the solid is called a *prismoid*, or pyramidoid.

[†] This rule is expressed arithmetically; the multiplication part may, however, be performed by the sliding rule.

at the bottom, and also of the breadth and thickness at the top: add the top and bottom breadths and also their thicknesses, and find the product of these two last sums; to which add the two former products, and the sum multiplied by one-sixth of the perpendicular, will give the solid content required.

Or,

To the breadth at the bottom, add half the top-breadth; and to the top-breadth, add half the bottom breadth; multiply the first of these sums by the bottom thickness, and the second by the thickness at the top: then, the sum of these products being multiplied by one-third of the perpendicular height, the product will be the solid content.

EXAMPLE.

Let the breadth at the bottom be 36 inches, and thickness 28; the breadth at the top 22 inches, thickness 16, and the perpendicular height 18 inches: required the solid content?

By Rule First.

Breadth at bottom	36	At top ········	22
Thickness at bott.	28	At top ········	16
	288		132
	72		22
First product····	100.8	Second product	352

Breadth

Breadth at bottom	36	Thick. at bott.	28
——— at top	22	——— at top	16
Sum	58	Sum	44
	44		
	232		
	232		
Product	2552		
First product	1008		
Second product	352		
Sum	3912		
One sixth height	3		
Solid content	11736		

By Rule Second.

Breadth at bottom	36	Half breadth at bot.	18
Half breadth at top	11	Breadth at top	22
Sum	47	Sum	40
Thickness at bott.	28	Thickness at top	16
	376	Second product	640
	94		
First product	1316		
Second product	640		
Sum	1956		
One-third height	6		
Solid content	11736		

PROBLEM

PROBLEM XIV.

To find the content of a cylindroid, which is a solid, whose bases are parallel ellipses.

RULE.

With the given transverse and conjugate diameters of each of the bases, and the perpendicular height, proceed as directed in last problem. Then, the product, thus obtained, being multiplied by .7854, will give the content of the cylindroid.

EXAMPLE.

Let the transverse diameter at the bottom be 48 inches, and the conjugate diameter 34 inches: those at the top 42 and 30 respectively; and the height 36 inches: required its solid content?

By Rule First.

Trans. diam. at bot.	48	Trans. diam. at top	42
Conjugate ······	34	Conjugate ····	30
	192	Second product	1260
	144		
First product ····	1632		
Trans. diam. at bot.	48	Con. diam. at bot.	39
—— —— at top	42	—— —— at top	30
Sum ············	90	Sum ··········	64
	64		
Product ········	5760		

Product ········ 5760
First product ···· 1632
Second product ·· 1260

Sum ············ 8652
One-sixth height ·· 6

Product ······· 51912; which being
Multiplied by ·· .7854 the product,

is······ 40771.68 the solid content of the cylindroid.

By Second Rule.

Trans. diam. at bot.	48	Half··········	24
Half trans. at top··	21	Trans. at top··	42
Sum ············	69	Sum··········	66
Conjugate at bot.··	34	Conjugate at top	30
	276	Second product	1980
	207		

First product ···· 2346
Second product ·· 1980

Sum············· 4326
Multiplied by $\frac{1}{3}^{d}$ h^{t}. 12

Product········ 51912 the same as before.

PROBLEM

PROBLEM XV.

Given the length of the keel of a ship, and the extreme breadth, to find the tonnage.

RULE.

Set 1 on B, to the length of the keel on A; and opposite to the breadth on B, is a certain number on A.

Now set 188 on B, to the above found number on A; and opposite to the breadth on B, is the tonnage required on A.

EXAMPLES.

I.

The length of the keel is 68 feet, and extreme breadth 22 feet; required the tonnage?

Set 1 on B, to 68 on A; and opposite to 22 on B, is 1500 on A. Now set 188 on B, to 1500 on A; and opposite to the breadth 22 on B, is 175 nearly on A, the tonnage required.

II.

Let the length of the keel be 86 feet, and extreme breadth 28 feet; required the tonnage?

Set 1 on B, to 86 on A; and opposite to 28 on B, is 2400 on A. Now set 188 on B, to 2400 on A; and opposite to 28 on B, is 358½ tons on A.

REMARK.

The tonnage may be more easily found by the following rule:

Set 13¾, or rather 13.71 *, on D, to the length of the keel on C; then opposite to the extreme breadth on D, is the tonnage on C.

EXAMPLE.

Let the length of the keel be 78 feet, and extreme breadth 24½ feet; required the tonnage?

Set 13.71 on D, to 78 on C; then opposite to 24½ on D, is 249 on C, the tonnage required.

By a sliding rule of a late construction the tonnage may be found as follows:

Set 1 on A, to 94 on D; now bring the half breadth on C, to the breadth on D; and opposite to the length of the keel on A, is the tonnage required on B.

EXAMPLE.

Let the length of the keel be 70 feet, and extreme breadth 24 feet; required the tonnage?

Set 1 on A, to 94 on D; then bring the half breadth 12 on C, to the breadth 24 on D; and opposite to the length 70 feet on A, is 214½ nearly, the tonnage on B.

SCHOLIUM.

The following method of taking the dimensions

---

* The number 13.71 is the tonnage point, and may be found on the rule by setting 10 at the middle of C, to 10 on D; then opposite to 188 (equal to twice 94) on C, is 13¾ nearly on D; or it is the square root of 188.

sions for the tonnage, is that which is used in the King's service; according to act of parliament.

I.

When the ship is lying dry.

Measure the length from the main stern-post, along the rabbet of the keel, to a line perpendicular thereto, from the fore part of the main stem, under the bowsprit; from which subtract three-fifths of the extreme breadth, exclusive of any sheathing or doubling; and the remainder is to be esteemed the length of the keel for ascertaining the tonnage.

II.

When the ship is afloat.

Measure the nearest distance, at the load water-mark, between the after part of the stern-post, and a plumb-line suspended from the stern; which subtracted from the length, measured from the point on the stern, where the plumb-line was suspended, to a point perpendicularly above the load-water mark, at the fore-part of the main stem, the remainder is the extreme length of the ship; from which subtract three-fifths of the extreme breadth, measured from outside to outside of the plank, exclusive of sheathing or doubling, and also one-fourth of the depth of the load water line abaft, and the remainder will be the length of the keel for tonnage; with which, and the extreme breadth, the tonnage is to be found as before.

EXAMPLES.

I.

Let the measured length from the after part of the post, along the rabbet of the keel, to a perpendicular, from the fore part of the stem, immediately under the bowsprit, be 127.8 feet, and the extreme breadth 32 feet; required the tonnage?

Set 5 on the line of numbers on the slide, to 3 on the first line of numbers; then opposite to the breadth 32 on the slide, is 19.2 on the line A: which subtracted from the measured length 127.8, the remainder 108.6, is the length of the keel for tonnage. Now set 13.7 on D, to 108.6 the length of the keel on C; then opposite to 32, the extreme breadth on D, is 591, the tonnage on C.

II.

Let the length between the after part of the post, at the load water-line, and a plumb-line suspended from the stern, measured horizontally, be 15 feet; the length upon deck from the point at the stern, where the plumb-line was suspended, and a point perpendicularly above the load water-mark at the fore part of the stem

198 feet: the extreme breadth 60 feet, and height of the load water-line abaft, 24 feet; required the tonnage of the ship?

Length, from the aft-side of the post, at the load water-line, to a perpendicular from the stern ··········	15 feet
Three-fifths of 60 feet, the extreme breadth ····················	36
One-fourth of 24 feet, the depth of the load water-line at the post ··	6
Sum ·························	57
Length measured upon deck······	198
Length of the keel for tonnage ··	141

Now, set 13.7 on D, to 141, the length of the keel on C, and opposite to 60, the extreme breadth on D, is 2700, the required tonnage on C.

SECTION V.

CONTAINING

THE PROPERTIES OF THE PLATONIC BODIES.

A regular solid is that which is contained by like, equal, and regular plane figures, and whose solid angles are all equal: of these there are only five, and they are called *Platonic* bodies from the ancient philosopher Plato. By the following table, the surface, solidity, &c. of any of these bodies may be very easily found:

Name.	Nº of Sides.	Th side of the Platonic body being 1.	
		Surface.	Solidity.
Tetraedron	4	1.7320508	0.1178513
Hexaedron	6	6.0000000	1.0000000
Octaedron	8	3.4641016	.04714045
Dodecaedron ..	12	20.6457788	7.6631189
Icosaedron	20	8.6602540	2.1816950

The diameter of a sphere being 1, the side of one of the Platonic bodies that

Name.	may be inscrib. in the Sphere, is	maybe circums. about the sph. is	is = to to the Sphere is
Tetraedron	0.816497	2.44948	1.64417
Hexaedron	0.577350	1.00000	0.88610
Octaedron	0.707107	1.22474	1.03576
Dodecaedron ..	0.525731	0.66158	0.62153
Icosaedron	0.356822	0.44903	0.40888

PROBLEM I.

To find the surface and solidity of a Platonic body.

RULE.

Multiply the number under *surface* corresponding to the given body, by the square of the diameter, the product will be the superficial content; and the number under *solidity* multiplied by the cube of the diameter will give the solid content required.

EXAMPLE.

Required the superficial and solid content of a dodecaedron, whose side is 10 feet?

Tabular surface	-	20.6457788	- solidity	-	7.6631189
Square of 10	=	100	- cube	-	1000
Surface	-	2064.57788	- solidity	-	7663.1189

PROBLEM II.

Given the diameter of a sphere, to find the side of a Platonic body that may be inscribed in the sphere circumscribed about it, or, that is equal to the sphere.

RULE.

Multiply the corresponding number in the table by the diameter of the sphere, and the product will be the side required.

EXAMPLE.

Required the side of an octaedron that may be circumscribed about a sphere, whose diameter is 20 inches?

Tabular number	1.22474
Given diameter	20
Side required	24.49480

The converse, &c. of these problems may also be easily resolved by means of this table.

CHAP.

CHAP. II.

CONTAINING

THE DESCRIPTION

OF THE

SHIP-CARPENTER's SLIDING RULE,

COMMONLY CALLED

MUNGO MURRAY's RULE;

AND

ITS USE IN THE CONSTRUCTION OF MASTS, YARDS, CAPSTAN-BARRELS, &c.

SECTION I.

CONTAINING

THE DESCRIPTION AND USE

OF THE

LINES UPON THE FIRST SIDE OR FACE OF THE RULE.

THIS rule is usually two feet long, having a joint in the middle, and a slide, as in the common sliding rule already described.

THERE are four lines of numbers, two upon the slide, marked B and C; one above the slide, marked A, and one below, marked D. This last line is inverted,* and is so disposed, that 12 on it is exactly opposite to 12 on the upper line of numbers on A; and, therefore, since the interval between 1 and 12, is greater than that between 12 and 100, the beginning of the inverted line extends beyond the termination of the line A; hence, the division representing 1.44 is at the right hand extremity of the inverted line, and 144 at its left.

---

* On some Carpenter's rules, the line D is the common girt line.

The superficial content is found by the two upper lines of numbers, A and B, exactly in the same manner as by the common rule before described. But the operation for finding the solid content of timber, stone, &c. is much easier, and is as follows:

PROBLEM.

Given the length in feet, and the breadth and thickness in inches, to find the solid content in feet.

RULE.

Set the breadth on the line C on the slide, to the thickness on the inverted line of numbers D; then opposite to the length in feet on the upper line A, is the solid content in feet on B.

EXAMPLES.

I.

Let the length of a log be 40 feet, breadth 21 inches, and thickness 16 inches; required the solid content?

Set 21 to 16 on the lines C and D; then opposite to the length, 40 on A; is the solid content 93⅓ feet on B.

The reason of this will appear obvious by attending to the method of performing the question by two operations on the lines A and B, as follows:

Set 16 on B, to 12 on A; then opposite to 21 on B, is a fourth number, 28 on A.

Again, set 28 on B, to 12 on A; and opposite to 40 on A, is the content 93⅓ on B.

Since, in the first operation, 16 on B is opposite to 12 on A, it will also be opposite to 12 on the inverted line D, and 12 on C will be opposite to 16 on D, the intervals between these points being equal; and because the interval between 12 and 21 on the line A, is equal to the interval between 16 on C and 21 on D; therefore, in the second operation, when the fourth term 28 on the slide is made to coincide with 12 on A, the point 21 on the line C, will coincide with 16 on the inverted line D; and the solid content 93⅓ feet, will be found upon B, opposite to the given length, 40 on A.

II.

Let the length of a log be 58 feet, breadth 32 inches, and thickness 21 inches; required the number of cubic feet it contains?

Set

Set 32 on C, to 21 on D; then opposite to the length 58 on A, is the solid content 270 feet on B.

III.

Let the length of a log be 27 feet, breadth 19 inches, and thickness 13 inches; required the solid content?

Set 19 on C, to 13 on D; and opposite to 27 on A, is 46 feet and above a quarter on B, the solid content required.*

DESCRIPTION
OF THE
EIGHT, TEN, AND TWELVE SQUARE LINES.

These lines, although employed in constructing regular polygons of wood, of eight, ten, and twelve sides; yet they have the denomination *square* annexed to them; and are, therefore, called the *Eight*, *Ten*, and *Twelve square Lines.*

The eight square lines are for making masts and yards; and the ten and twelve square lines are for making barrels of capstans. The eight square lines are on the inner edge of each leg of the rule, and are marked M, and E, signifying *middle, and edge;* and the ten and twelve square lines are upon the same side of the rule as the slide, but on the other leg, except the last line, of which there is a part on each leg. These lines for the ten and twelve squares are five in number, and are titled as follows: 10 Sqr. B. S.; 10 Sqr. S. S.; 12 Sqr. 2 S.; 12 Sqr. 4 S.; and 10 L. D. Sqr. In the ten square lines, B. S. signifies *broadest side*; S. S. *smallest side*; and L. D. the *length* of the *diameter* passing through any two of the opposite corners, or angular points of the polygon. Also, in the twelve square lines, 2 S. signifies the breadth to be laid off from the middle line upon any *two* of the opposite *sides*; and 4 S. that to be laid off from the middle line on all the four *sides*. The four first of these lines begin immediately after their titles; and the fifth or last begins at the left hand end, and terminates at the extremity of the other leg.

---

* The various operations in multiplication, division, proportion, square root, &c. are also easily performed by the inverted line of numbers.

USE OF THE EIGHT SQUARE LINES.

The wood being previously squared, that is, of the same breadth and thickness, and the sides perpendicular to each other; then let lines be drawn along the middle of each side. Now from the line marked M, on the inner edge of the rule, take the number answering to the breadth or thickness in inches, and lay it off both ways from each middle line; and through these points let lines be drawn parallel to the middle line; then the angular pieces between these lines being cut off, the wood will be eight-square, as required.

Or the number answering to the breadth or depth being taken from the line marked E, and laid off from the edge of the wood, will give the same points as before.

In making a mast, the log is to be squared, and lines struck along the middle of each side. The places of the partners, quarters, &c. are to be marked; then the numbers from the line M, answering to the several breadths at the partners, quarters, &c. are to be taken and laid off from the middle line at these places respectively, and lines are to be struck between each adjacent pair of points.

In making a yard, the divisions answering to the thickness at the slings, quarters, and yard arm, are to be taken from the line M, and laid off from the middle line as before.

If the thickness of a yard at the slings be called 1; then the thickness at the first quarter will be expressed by the fraction $\frac{27}{28}$; that at the second quarter, by $\frac{9}{10}$; at the third by $\frac{7}{10}$; and at the yard arm by $\frac{4}{10}$. Hence, the diameter at the slings being given, the several diameters at the quarters and yard arm may be found by setting the denominator of the fraction to its numerator; and opposite to the diameter at the slings, is the corresponding thickness. Or, set the denominator of the fraction to the thickness at the slings, and opposite to the numerator is the required thickness.

EXAMPLE.

Let the diameter of a yard at the slings be 25 inches, required the several measures for the quarters and yard arm?

By proceeding as directed, the diameter at the first quarter will be found to be $24\frac{1}{21}$ inches, or 24 inches and one-tenth nearly; at the second quarter, 22½ inches; at the third quarter, 17½ inches; and, at the yard arm, 10 inches.

REMARK.

The number to be laid off from the middle line may be found independent of the line M, as follows:

Set 100 on B, to 20.7 on A; then opposite to the diameter on B, is the number required on A.*

Thus, let the diameter be 16 inches, to find the number to be laid off on each side of the middle line.

---

* Or multiply the number .2071 by the diameter, and point off four figures to the right hand.

Set 100 on B, to 20.7 on A; then opposite to 16 on B, is 3.3 on A.

The number to be laid off from the edge may be found as follows:

Set 100 on B, to 29.3 on A; and opposite to the breadth or thickness on B, is the number to be laid off from the edge on A.*

Thus, let the breadth be 24 inches, required the number to be laid off from the edge?

Set 100 on B, to 29.3 on A; and opposite to 24 on B, is 7.03 on A, the required number.

USE OF THE TWELVE SQUARE LINES.

The wood being of an equal breadth and thickness, and the sides perpendicular or square to each other, take the number answering to the breadth or depth in inches from the line marked 12 Sqr. 2 S. and lay it off from the middle both ways on any two opposite sides of the wood, and draw lines through these points parallel to the middle line. Also, lay off, from the middle line on each of the four sides, the extent from the line marked 12 Sq. 4 S. answering to the breadth or thickness in inches, and draw lines through these points parallel to the middle line. Now, cut off the angular pieces of wood between the lines 2 S. and 4 S; and upon this new side lay off, from the line 4 S. double the measure from the line 12 Sqr. 4 S; and the angular piece of wood between this line and the adjacent line 4 S. being cut off, will form the 12 square required.

---

* Or, multiply the constant number .2929 by the breadth or thickness, and the product, four figures being pointed off towards the right hand, will be the number required.

REMARK.

The number to be laid off from the middle line may be found in the following manner.

Set 100 on the line B, to 13.4 on A; and opposite to the breadth on B, is the number on A, answering to that from the line marked 12 Sqr. 4 S.*

And set 100 on B, to 28.9 on A; and opposite to the breadth on B, is the number on A, answering to the measure from the line 12 Sqr. 2 S.†

---

* Or, multiply the constant number 134 by the breadth, and the product, three figures being pointed off to the right hand, will be the measure to be laid off from the middle line on each of the four sides.

† Multiply the constant number 2887 by the breadth, and the product, four figures being pointed off to the right hand, will be the measure to be laid off from the middle line on two of the opposite sides.

Thus, let the diameter be 18 inches, required, the measures to be laid off from the middle line?

Set 100 on B, to 13.4 on A; and opposite to 18 on B, is 2.4 inches, the measure answering to that from the line 12 Sqr. 4 S.

Also, set 100 on B, to 28.9 on A; and opposite to 18 on B, is 5.2 inches, the measure to be laid off on any two of the opposite sides.

USE OF THE TEN SQUARE LINES.

The wood intended for a ten-square barrel must be broader than it is thick. When the thickness is given, the breadth may be found by taking the corresponding number from the line marked 10 L.D. Sqr. Thus, if the thickness is 9 inches, then the breadth will be the extent between the beginning of the line 10 L.D. Sqr. and the division 9, which will be about 9½ inches.

Now, the wood being made to the proper breadth and thickness; take, from the line marked 10 Sqr. B. S. the number answering to the thickness in inches, and lay it off both ways from the middle line on each of the broadest sides, and through these points draw lines parallel to the middle line. Again, take from the line marked 10 Sqr. S.S. the number answering to the thickness in inches, and lay it off both ways from the middle line on each of the smallest sides, and let lines be drawn through these points parallel to the middle line. Now cut off the angular pieces between these lines; and, upon the sides thus formed, lay off, from each of the lines drawn on the broadest side, the interval between these lines, which is one of the sides of the ten square; then through these points draw lines parallel to the former lines, and the angular pieces between these lines and the middle lines on the smallest sides being cut off, the wood will then be ten squared.

REMARK.

The thickness or smallest side being given, the breadth may be found by setting 20 on B, to 21* on A; and opposite to the thickness on B, is the breadth on A;† and, conversely, if the breadth be given, set 21 on B, to 20 on A; and

---

[*] Or, more exactly, 21.029244.

[†] This may be done without the assistance of the rule as follows: Multiply the number 1.0514622, the secant of 18° by the smallest side, and the product will be the breadth.

and opposite to the breadth on B, is the thickness on A.*

In order to find the measure to be laid off from the middle line on each of the broadest sides, set 100 on B, to 30.9 on A; and opposite to half the breadth on B, is the measure required on A, that is, the measure from the line 10 Sqr. B.S.†

And to find the measure to be laid off from the middle line of the smallest side, set 100 on B, to 44.9 on A; and opposite to half the breadth on B, is the measure on A, to be laid off from the middle line of each of the smallest sides, and which will answer to the measure from the line 10 Sqr. S.S.‡

SECTION II.

Of the lines for ascertaining the lengths, and diameters of masts, yards, &c.

Upon the inner side of the slide are divisions for

---

[*] Multiply the decimal .9510565, the sine of 72°, or, for common practice, 951 by the breadth, and the product will be the smallest side.

[†] Or, multiply the decimal .309, the sine of 18°, by half the broadest side, and the product will be the quantity to be laid off from the middle line of that side.

[‡] Or, multiply the decimal .449, which is the sine of 72° —Versed sine of 72° × tangent of 36°, by half the breadth, and the product will be the measure required.

for ascertaining, when compared with the line of numbers, the lengths of masts, yards, and the diameters of each.

This line contains three principal divisions: The first division, or that for masts, contains the following subdivisions, viz.

Brh. signifying the extreme breadth of the ship.

m.	main-mast.
f.	fore-mast.
z.	mizen-mast.
b.	bowsprit.
mt.	main-top-mast.
ft.	fore-top-mast.
zt.	mizen top-mast.
mg.	main top-gallant-mast.
fg.	fore top-gallant-mast.

In the second or middle division.

G.D. signifies Gun deck.

m.	main-yard.
f.	fore-yard.
z.	mizen-yard.
mt.	main top-sail-yard.
ft.	fore top-sail-yard.
mg.	main top-gallant-yard.
zt.	mizen top-sail yard.
fg	fore top-gallant-yard

And the third and last division, for finding the diameters of masts and yards, contains the following subdivisions, viz.

Beginning at F. L. signifying feet in length.
b. .... bowsprit.
lm. .... lower masts.
ly. .... lower yards.
zm. ...... mizen-mast.
ty. .... top-sail-yards.
gy. ...... top-gallant-yards.
And zy. ...... mizen-yards.

In ships of war, the length of the main mast is adapted to the extreme breadth; and the lengths of the other masts are deduced from that of the main mast, agreeable to received proportions. The lengths of the yards are adapted to the length of the gun-deck; and the diameters of each are proportioned to the length of the mast or yard respectively. The use of these lines will be obvious from the following problems:

PROBLEM I.

Given the extreme breadth of a ship, to find the lengths of the masts, top-masts, &c.

RULE.

Take out the slide, and turn it so that the inner side may be uppermost, and the extremity marked Brh. toward the right hand. Now move the slide until the division Brh. be exactly opposite to the extreme breadth in feet on the upper line of numbers A; then opposite to each of the divisions m. f. z. &c. on the slide, are the corresponding lengths of the several masts, topmasts, &c. in yards respectively.

EXAMPLE.

Let the extreme breadth be 30 feet, required the lengths of several masts, topmasts, &c.

Set the first division Brh. on the slide, to the extreme breadth 30 on the line A; then opposite to m. is 22½ yards, the length of the mainmast on A. and opposite to

f. is	20⅓	fore mast.
z.	19½	mizenmast.
b.	14	bowsprit
mt.	13¾	main top-mast.
ft.	12⅔	fore top-mast.
zt.	10	mizen top-mast.
mg.	7	main top-gallant-mast.
fg.	6½	fore top-gallant-mast.

PROBLEM II.

Given the length of the gun-deck to find the lengths of the yards.

RULE.

Set GD on the slide, to the length of the gun-deck on the line A; then opposite to the several divisions on this part of the slide are the lengths of the yards in feet.

EXAMPLE.

Let the length of the gun-deck be 138 feet, required the lengths of the yards?

Set GD on the slide, to 138 on the line A; then opposite to the several divisions on the slide are the corresponding lengths of the yards on A, viz.

Op. to m. is 80 feet, the length of the main-yard.
f. 70 ······ fore yard.
z. 67 ······ mizen yard.
mt. 58 ······ main top-sail-yard
ft. 50½ ······ fore top-sail-yard.
mg. 40 ······ main top-gallant-yard.
zt. 38 ······ mizen top-sail-yard.
And fg. 35 ······ fore top-gallant yard.

PROBLEM III.

Given the length of a mast or yard to find the diameter of the mast at the partners, or of the yard at the slings.

RULE.

Set the division F.L. on the slide, to the length, in feet of the given mast or yard on the line A; and opposite to the division representing the mast or yard, on the slide, is the diameter in inches on the line A.

EXAMPLES.

I.

Let the length of the main-mast be 80 feet, required its diameter at the partners?

Set F.L. on the slide, to 80 on A; then opposite to *lm* on the slide, is the diameter 25 inches on A.

II.

The length of a main top-mast is 60 feet, required its diameter?

Set F.L. on the slide, to 60 on A; and opposite to *mt* on the slide, is 18¼, the diameter on A.

III.

The length of a main yard is 70 feet, required its diameter at the slings?

Set F.L. on the slide, to 70 on A; and opposite to *ly* on the slide, is 17 inches, the diameter of the yard at the slings.

SECTION II.

CONTAINING

THE DESCRIPTION AND USE

OF THE

LINES UPON THE SECOND SIDE, OR FACE OF THE RULE.

Upon the other side of the rule, with respect to that in which is the slide, besides a line of inches, are two lines of equal parts, one on each leg. Each principal division in these lines, which is about three quarters of an inch, is divided into twelve equal parts: hence, each primary division may represent feet, and the intermediate divisions will represent inches. By these lines, a scale may be laid down of any proposed length; a given line may be divided into any assigned number of equal parts; and the various operations in proportion may be performed. These will be illustrated by the following problems:

PROBLEM I.

To lay down a scale of any given length.

RULE.

Take the given length which is to represent twelve feet on the scale, and open the rule until the divisions representing twelve on each leg be exactly equal to the above extent; then the intervals between the intermediate principal divisions will represent feet, and those between the subdivisions, feet and inches.

EXAMPLE.

Let it be required to lay down a scale so that $\frac{5}{8}$ths of an inch may represent one foot?

As $\frac{5}{8}$ths of an inch is to represent one foot, then $\frac{5}{8}\times 12=7\frac{1}{2}$ inches, will represent 12 feet. Now take $7\frac{1}{2}$ inches from the scale of inches, and open the rule until the divisions marked 12, on the nearest parts of the lines be equal to this extent; then the extent between 1 and 1, will represent 1 foot; that between 2 and 2, will be 2 feet; between 4.3 and 4.3 will represent 4 feet, 3 inches, &c.; and hence the scale may be laid down upon paper.

PROBLEM II.

To divide a given line into any proposed number of equal parts.

RULE.

Make the given line a parallel extent[*] between the divisions on the opposite legs, representing the number of parts into which the line is to be divided; then lay off the intermediate extents upon the line, and it will be divided accordingly.

EXAMPLE.

Let it be required to divide a line 4½ inches long into 7 equal parts?

Take 4½ inches from the scale of inches, and make it a parallel extent between 7 and 7 on the line of equal parts; then the several extents between 1 and 1, 2 and 2, &c. being laid off upon the line, will divide it into 7 equal parts.

---

[*] A *parallel extent*, is an extent measured in a direction across, between any two correspondent divisions; and an extent measured along the line from the center is called a *lateral extent.*

PROBLEM III.

To find a fourth proportional to three given numbers.

RULE.

Take the second term from the line, and make it a parallel extent between the divisions representing the first term; then the parallel extent between the divisions representing the third term, being applied to the line from the beginning, will give the fourth term.

EXAMPLE.

Required a fourth proportional to the numbers 14, 6, and 9?

Take 6 from the line, and open the rule until the extent between 14 and 14 is exactly equal to 6; then the extent between 9 and 9 being laid off from the center, will measure 3 and $\frac{12}{14}$ths nearly.

Of the lines for determining the thickness of masts and yards at the quarters, &c.

Within the above lines of equal parts are four lines on each leg of the rule for making masts and yards. These lines begin at the right hand extremity, and, if produced, would intersect in the centre of the joint.

The first or innermost line is for making masts. At the right hand extremity it is marked P. signifying *partners*. The 1st, 2d, and 3d quarters are marked. Also hs. is for *hounds;* and, at the left hand extremity, hd. for *mast-head*.

The second, or next line, marked B. for *bed*, at the right hand end, is for making bowsprits. The quarters are marked as before; and at its other extremity are the letters B.C. signifying the *breadth* at the *cape*.

The third line is for making yards. At the right hand extremity is marked S. signifying *slings*. The quarters are marked; and at the other end are the letters Y. A. for *yard-arm*.

The fourth, or outermost line, is for a mizen yard: upon which, at the right hand end, S. stands for *slings;* L. A. for *lower arm*. The use of these lines will be obvious from the following problem:

PROBLEM IV.

Given the length of a mast, and the diameter at the partners to make it.

RULE.

Let a line be struck along the middle of two of the opposite sides of the log intended for the mast; upon which mark the place of the partners and hounds, and divide this interval into four equal parts. Now with a pair of compasses take half the breadth at the partners, and open the rule until the distance between the points P. and P. at the end of the rule, be exactly equal to this extent, then take the extent between the dots at the first quarter on the rule, and lay it off at the first quarter on the mast, from the middle line on each side. In like manner, lay off the other quarters, the hounds and mast head; and let lines be struck between the points, and hew off the wood accordingly. Proceed, in like manner, with the other two sides; then eight square it by the preceding rules, and finish it off as usual.

A yard is constructed nearly in the same manner. The diameter at the slings is to be made a parallel extent between S. and S. on the rule, and the quarters and yard-arm are to be taken from the rule, and laid off from the middle line as before.

Upon one of the outer edges of the rule, the foot is divided into 100 equal parts; and upon the other edge are two lines of numbers, one a single line, and the other a triple line. By these lines, the cube root of any given number is found by inspection, and conversely. Thus, find the given number on the triple line, and opposite to it on the single line is the root required.

In this operation, it must be observed, that if the number whose root is required consists of 1, 4, 7, &c. places of figures, it is to be found on the first part of the triple line; if it consists of 2, 5, 8, &c. places of figures, it is to be found on the middle part; and if it has 3, 6, 9, &c. places of figures, it will be found on the last part: opposite to which, on the single line, is the root required. If the number of figures in the proposed cube be three or under, the root will consist of one figure; from three to six places inclusive, the root will consist of two figures; from six to nine inclusive, of three figures, &c.

This line is also useful in finding the dimensions of a solid similar to one given. For this purpose, however, a pair of compasses must be used, as follows: extend the compass from the content of one solid to that of the other, on the triple line; and this extent being laid the same way from a side of the first solid on the single line, will reach to the corresponding side of the second solid; and conversely.

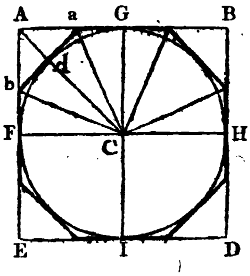
SECTION III.

CONTAINING
THE INVESTIGATION
OF THE
METHODS OF CONSTRUCTING
THE
EIGHT, TEN, AND TWELVE SQUARE LINES.

I.

OF THE EIGHT SQUARE LINES.

Let A B D E be the section of a log, to be formed into a regular polygon of eight sides.



From the center C, with the radius C F, or C G, describe the circle G F I H, and draw the diameters F H, G I, parallel to the sides of the square. Divide each quadrant into four equal parts; and from the center, through each of these points, except the middle in each quarter, draw lines, meeting the sides of the square in the points a, b; join these points by the straight line a b, and the figure will be a regular octagon. Hence, G a is the measure to be laid off from the middle line, and A a, or A b, that from the edge, in order that the portion A b a may be cut off, to reduce the square log to an eight-sided figure.

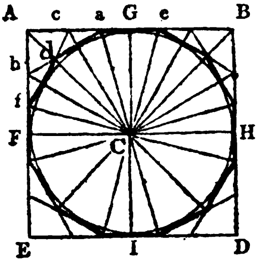
In the right angled triangle, C G a, the angle G C a, being the fourth part of a quadrant, or 90 degrees, is equal to 22° 30′; therefore, G a, is the tangent of 22° 30′, to the radius C G; and G A is equal to C G; hence, A a, is the difference between the radius, and the tangent of 22° 30′. Now, the radius being 1, the tangent of an arch of 22° 30′ is equal to the square root of 2—1. or .4142=G a. Also, A a, or the measure to be laid off from the edge, is equal to the difference between A G and G a, equal to 1—.4142,=.5858. Hence, a scale may be laid down, from which the measures may be taken, for any given diameter, or side of the square, expressed in inches, by multiplying the constant decimal number .4142, by half the given number of inches, when the measure is to be laid off from the middle line; or, by using the decimal .5858, when the measure is to be laid off from the edge.

In order to illustrate the above, let it be required to lay down the division on the rule, answering to a log 28 inches square.

The half of 28 is 14, which, multiplied by the decimal .4142, the product 5 inches and 8-tenths nearly, being taken from a scale of inches, will reach, on the line M, from the beginning of that line, to the division marked 28 Also, 14 being multiplied by the decimal number .5858, and the product 8 inches, and 2 tenths, taken from a scale of inches, and laid off from the beginning of the line marked E, will reach to the 28th division. In like manner, the other divisions may be found. Or, this space may be divided into the corresponding number of equal parts, and extended at pleasure.

II.

OF THE TWELVE SQUARE LINES.



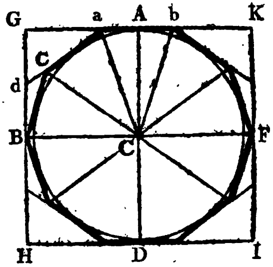
In the section of the log ABD E, inscribe the circle G H I F, and draw the diameters F H and G I parallel to the sides of the square A B and A E; also draw the diagonals A D and B E. Divide the quadrant G F, into six equal parts, and from the center draw lines through these points of division, meeting the sides G A and A F, in the points a, c, b, f; join a b, and f c, which will intersect each other in the point d, in the diagonal A D; and a d, d f, will be two of the sides of the figure. By proceeding in this manner in the other quarters of the circle, the twelve-sided figure will be completed.

Since the log of wood is to be formed into a regular polygon of twelve sides, the angle at the center, a C e, being the third part of a quadrant, or 90 degrees, is, therefore, equal to 30°; and, consequently, G C a, is 15°. Now, G a, or F f, which is the measure from the line marked 12 Sq. 4 S, is equal to the tangent of 15° to the radius C G; which being called 1, the tangent of 15° is equal to 2—the square root of 3; or to the decimal .268 nearly. Also, G c, or F b, the measure from the line 12 Sq 2 S. is the tangent of 30° which is equal to the square root of one-third; or, to the decimal .577.

To find, therefore, the point on the line marked 12 Sq. 4 S. answering to any given side of a square, as 28 inches, the half of the given number, viz. 14 inches, being multiplied by the decimal .268, the product is 3.75; which being taken from a scale of inches, will reach from the beginning of the line 12 Sq. 4 S. to the point representing 28. Also, the decimal .577 being multiplied by 14, the half of the given side of the square, and the product 8.078, taken from a scale of inches, will reach from the beginning of the scale, marked 12 Sq. 2 S. to the division representing 28. As the divisions on these lines are equal, the intermediate ones may be easily found; and the lines may be extended at pleasure.

III.

OF THE TEN SQUARE LINES.



Let C A be half the thickness of the log; then from the center C, with the distance C A, describe a circle; and through C, draw B F at right angles to C A: Now since the figure is to contain ten equal sides, the angle a C b, at the center, will be equal to one-tenth of 360° = 36°, and the half, A C a, will be = 18°. Make, therefore, the angle A C a equal to 18°, or A a the tangent of 18°, to the radius A C; and join C a: then, A a, or A b, will be the measure to be laid off from the middle line of the breadth, or the division from the line 10 Sqr. B S. and C a will be half the broadest side; which, to the radius C A, or 1, will be 1.05, the secant of 18°. Make C B, C F, each equal to C a, and complete the parallelogram G H I K; then C B, or C F will be half the breadth.—Bisect the angle B C a, by the straight line C c; which make equal to C a; join a c, and produce it, till it meets B G in d; and B d will be the measure to be laid off from the middle line of the thickness; or the division from the line 10 Sqr. S S.

Since A a, is equal to .325 nearly, the tangent of 18°, half the thickness being taken for the radius or 1; therefore, the above quantity taken from a scale of inches, and laid off from the middle line on the broadest side, will give the point representing one division, or inch, to be laid off from that line: but in order to avoid taking the halves of the dimensions, and to adapt the scale to the whole thickness, the above quantity is made to stand for two divisions; and hence, the line is easily constructed. For example, if the division representing 20 inches on the line marked 10 Sqr. B S, is required. Then multiply .325 by 10, the half of 20, the product 3.25, being taken from a scale of inches, and laid off from the beginning of the line 10 Sqr. B S, will give the point representing the division marked 20. Hence, this interval may be subdivided, and the line extended at pleasure.

Again, to find B d, the quantity, from the line 10 Sqr. S S, to be laid off from the middle line of the thickness, or least side. In the right angled triangle, aGD, there are given, aG, equal to $AG - Aa =$ secant of $18^\circ -$ tangent of $18^\circ = 1.0514622 - .3249197 = .7265425$, which is also $=$ tangent 36°; and the angle $Gad = 180^\circ - CaA + Cac = 180^\circ - 144^\circ = 36^\circ$. Hence, Gd, which is the square of the tangent of 36°, will be found to be .52786; therefore, $Bd = BG - Gd = 1.00000 - .52786 = 47214$. Whence .472 being taken from a scale of inches, and laid off from the beginning of the line marked 10 Sqr. S S, will give the point representing the division 2, for the reason assigned above; and 4.72 inches, laid off from the beginning of the line, will give the point representing 20; and hence, the line may be subdivided, and continued at pleasure.

CHAP. III.

CONTAINING

THE DESCRIPTION OF THE GAUGING RULE,

AND ITS USE,

Applied to Gauging and Ullaging of Casks, &c.

SECTION I.

INTRODUCTION.

GAUGING, is that part of mensuration, by which the capacity of a vessel is ascertained in some fixed measure, as *gallons*, *bushels*, &c.

According to act of Parliament, a wine gallon contains 231 cubic inches; a beer gallon 282; and a malt bushel contains 2150.42 cubic inches. Hence, the content of any vessel, in cubic inches, being found, by the rules laid down in Chap. I.; its capacity, or measure in gallons or bushels, is found by dividing the content of the vessel, expressed in cubic inches, by the proper divisor; that is, the content is to be divided by 231 to reduce it to wine gallons; by 282, to beer gallons; and by 2150.42 to reduce it to malt bushels. This, therefore, is so obvious, as to render it unnecessary to repeat any of these operations.—The principal part remaining to be exemplified is that of gauging, and ullaging casks; which being generally performed by a rule, it will, therefore, be necessary to give a brief description of that instrument.

SECTION II.

CONTAINING

THE DESCRIPTION OF THE GAUGING RULE.

The gauging rule is commonly made of box wood; being one foot long; one inch and one-tenth in breadth; and eight-tenths of an inch thick. It consists of five parts, viz. a stock and four sliders.

Upon the first face or side of the rule, is a single line of numbers marked A, at the right hand extremity. On this line, M, B, signify *malt bushels*, and A, *ale gallons*.—Upon the slide, are two lines of numbers, marked B, at the left hand; they are similar to the former, with this difference only, that the division 95 terminates the right hand extremity of the rule. The lower line, or that on the stock, is marked M, D, signifying *malt depth*. This is an inverted line of numbers, and so graduated, the divisor 2150.4, for malt bushels, is at each end of the line, immediately opposite to 1, and 10, upon the line A; and, consequently, 10 on M D, is exactly opposite to the divisor M B, on the line A. The inside of the slide contains a line of inches, with lines for reducing the first and second varieties of casks to a mean diameter.

Upon the stock of the opposite side of the rule, is a line of numbers beginning at 1, and ending at 3.2, on the upper part; which is continued on the lower part of the stock from 3.2 at the left hand, and ends at 10. This line is marked D.——the points marked W. G. and A, G, on the upper part of the line, are the gauge points for *wine*, and *ale* gallons; and the divisions marked M. S. and M. R. are the gauge points for *malt square* and *malt round*:—There are two single lines of numbers on the slide marked B. At the division 231, W. is placed for *wine gallons;* and on 3.14, C. is marked to represent the circumference of a circle whose diameter is 1, or the area of a circle whose diameter is 2.—Upon the inside of the slide, are the square and circular gauge points for wine, and ale or beer gallons, malt bushels; and for green and dry starch per pound; hot and cold hard soap, green and white soft soap; flint, white and green glass, &c.

In order to illustrate the method of ascertaining these gauge points; let it be applied to any particular case, as that of green glass, of which a pound is found to contain about 12.18 cubic inches. Now, the square root of 12.18 is 3.49, the square gauge point; and 3.49 divided by the square root of .7854 gives 3.948, the circular gauge point for green glass; and in like manner, the gauge points for soap, starch, &c. are found.

One edge of the rule contains a line for finding the ullage of a lying cask, and is marked *Seg. ly*, this line is continued below the slide, and ends at 100: the slide contains two single lines of numbers. Upon the other edge of the rule is a line marked *Seg. St.* for finding the ullage of a standing cask; this line is continued below the slide, and ends at 100: and upon the slide are two single lines of numbers: the lines on both sides are marked C.—Upon the inside of the slides are the square and circular divisors, &c.

All the lines of numbers, being logarithmic scales, are constructed in the same manner as those upon the common sliding rule: see page 7. And the lines of segments may be constructed experimentally, as follows:

Let a cask be made nearly of the same figure as the middle frustum of a spheroid, its length being 45 inches, bung diameter 30, and head 24.2 inches, which will contain 100 ale gallons. Fill the cask with water, and place it so, that its axis may be horizontal: then, draw off one gallon, and, with the rod, find the depth of the remaining water at the bung.—Now, set the bung diameter on the slide, to 100 on the line of segments, and opposite to the wet inches, and parts of an inch, on the slide, mark the division 99, on the line of segments. Again, draw off another gallon, measure the wet inches, and opposite thereto on the slide, mark the division 98 on segments; and, in like manner, proceed until the line of segments is graduated.

The same process is to be adopted in constructing the line of *Segments Standing;* with this difference only, that the cask is to be so placed that its axis may be perpendicular to the horizon, and the rod is to be put in at the head of the cask.

The line of *segments lying* may also be constructed, by dividing the bung diameter into 100 equal parts, and finding the content of each section; and that of *segments standing*, by dividing the length of the cask into 100 equal parts, &c.

SECTION III.

CONTAINING

The Method of finding the Divisors, Multipliers, and Gauge Points used in Gauging.

In order to ascertain the number of gallons or malt bushels contained in any vessel, its content in cubic inches is to be found; which being divided by the proper divisor, will give the content in gallons or bushels. Instead, however, of dividing, if the reciprocals of the divisors be found, the content of the vessel, expressed in cubic inches, will be reduced to gallons, or malt bushels, by multiplication.—The reciprocal of 231, or 1 divided by 231, is .0043289, which is the multiplier for reducing the content of the vessel expressed in cubic inches to wine gallons. The reciprocal of 282 is .0035461, the multiplier for ale gallons; and the reciprocal of 2150.4 is .000465, the multiplier for malt bushels.

The gauge points on the rule for square figures, are the square roots of the above divisors. Thus, the gauge point for wine gallons is the square root of 231, which is 15.19. that for ale gallons is the square root of 282,=16.79; and the gauge point for malt bushels is 43.37, the square root of 2150.4.

In performing operations in gauging, by the rule, it is found convenient, particularly when the end of the slide is past the proper divisor, to have a second set of gauge points; these are found by taking the square roots of ten times 231 and 282; and that of one-tenth of 2150.4. Thus, the second gauge point for wine gallons is 48.06, the square root of 2310: for ale gallons it is 53.1, the square root of 2820; and the second guage point for malt bushels is 14.66, the square root of 215.04.

The diameter of a circle being 1, its area is .7854. and since circles are to each other as the squares of their diameters; if, therefore, the square of the diameter of a circle, in inches, be multiplied by .7854, the product will be the area of the circle in inches; which must be divided by the proper divisor to reduce it to gallons or bushels; but since .7854, and the divisors are constant quantities, it is better to reduce the vulgar to a decimal fraction; if, therefore, multipliers are wanted, .7854 is to be divided successively by 231, 282, and 2150.4. Now, 7854, divided by 231, gives .0034, the multiplier for wine gallons; and by 282, the quotient .002785 is the multiplier for ale gallons; and .7854 divided by 2150.4, gives .000365, the multiplier for malt bushels. If divisors are wanted for obtaining the same result; either the reciprocals of the multipliers, or the quotients resulting from the division of 231, 282, and 2150.4 successively, by .7854, will be the required divisors. Thus, 231 divided by .7854. the quotient 294.12 is the divisor for wine gallons: 282 divided by .7854, gives 359.05 the divisor for ale gallons, and 2150.4 divided by .7854, the quotient 2748 is the divisor for malt bushels.

The gauge points for circles are the square roots of the above divisors. Thus, the square root of 294.12 is 17.14, the gauge point for wine; the square root of 359.05 is 18.95, the gauge point for ale; and the square root of 2738 is 52.32, the gauge point for bushels.—The second gauge points are found by taking the square roots of ten times the two first divisors, and of one-tenth of the third divisor. Thus, the square root of 2941.2 is 54.22 the second gauge point for wine gallons; the square root of 3590.5 is 59.92, the second gauge point for ale gallons; and the square root of 273.8 is 16.54 the second gauge point for bushels. The gauge points being the square roots of the divisors are, therefore, the first term of each proportion.

SECTION IV.

CONTAINING

THE USE OF THE RULE IN CASK GAUGING.

Cask Gauging, is the art or method of ascertaining the content of a cask, in some known measure, as gallons, &c.

Casks, although their lengths, bung and head diameters are the same, may be of very different capacities, in consequence of the various degrees of curvature of the staves; and, therefore, for the more readily ascertaining their contents as nearly, and with as little trouble as possible, they are generally divided into four different forms called *varieties,** to one of which, any cask to be measured, is to be referred.

If the staves of the cask be much curved, it is considered to be the middle frustum of a spheroid. A cask of this form is said to be of the *first variety;* and contains more than any other cask whose length, bung, and head diameters are the same.

If the staves are straight between the bung and head of the cask, it is considered to be composed of the lower frustums of two equal cones: this form is called the *fourth variety;*

---

* Probably the decimal division would have been more accurate.

and contains less than any cask, whose length, and bung, and head diameters are the same.

Between these, two other varieties are introduced. That nearest to the spheroid, which is called the *second variety*, is taken to be the middle frustum of a parabolic spindle; and the other, or *third variety*, is considered to be the lower frustums of two equal parabolic conoids.

In order to find the content of a cask, it is previously reduced to a cylinder, by encreasing the head diameter. For this purpose lines are placed on the rule, adapted to the several varieties of casks, and are to be used as follows. Find the difference between the bung and head diameter on the line of inches, and opposite thereto, on the line answering to the variety of the cask, is a number, which added to the head diameter, the sum will be a mean diameter, or that of a cylinder of the same length and capacity as the given cask. Or if the difference between the bung and head diameters, be multiplied by .7 for the first variety; .65 for the second; .6 for the third; and .55 for the fourth variety; and the product added to the head diameter, the sum will be the mean diameter of the cask. In those casks where the difference between the bung and diameters, is less than six inches, it is customary to use the following multipliers, viz: .68 for the first variety; .62 for the second; .55 for the third; and .5 for the fourth variety.

It may be observed, that Jamaica puncheons are nearly the frustum of a spheroid; and are, therefore, generally considered to be of the first variety. Coniac pipes are, for the most part, of the second variety; and Madeira pipes, of the third variety; very few casks are of the fourth variety.

The head diameter, when measured upon the outside, is less than the inside measure; and must, therefore, be encreased by a small quantity; which, were the thickness of the head, the length, and the difference between the head and bung diameters of the cask known, might be pretty accurately determined; being a fourth proportional nearly to half the length of the cask, the thickness of the head, and the difference between the bung and head diameters. The usual allowance, however, for casks under 30 gallons, is three-tenths of an inch; between 30 and 50 gallons, four-tenths; and for casks exceeding 50 gallons, from five to six-tenths of an inch.

PROBLEM.

Given the length of a cask, its bung and head diameters, and the variety to which it belongs, to find its content.

RULE.

Find the mean diameter of the cask, by multiplying the difference between the bung and head diameters, by the multiplier, corresponding to the variety, and adding the product to the head diameter. Then set the length on C, to the gauge point on D; and opposite to the mean diameter on D, is the content of the cask on C.

EXAMPLES.

I.

Let the length of a cask of the first variety, be 21 inches, the bung diameter 19 inches, and the head 16 inches; required its content in beer gallons?

The difference between the bung and head diameter is 3 inches; which multiplied by .7, and the product 2.1 being added to the head diameter, the sum 18.1 is the mean diameter. Now set 21, the length on C, to the gauge point A. G. on D; and opposite to 18.1, the mean diameter on C, is the content, 19.2 gallons on C.

II.

The length of a cask, of the third variety, is 38 inches, the bung diameter 30, and the head diameter 22 inches; required its content in wine gallons?

Since the cask is of the third variety; the difference between the bung and the head diameters, viz. 8, is to be multiplied by .6; and the product 4.8, being added to 22 the head diameter, the sum 26.8 is the mean diameter. Now set the length on C, to the gauge point W. G. on D; and opposite to the mean diameter 26.8 on D, is the content of the cask, 92.8 wine gallons on C.

REMARK.

The content may be found by logarithms as follows. To twice the logarithm of the mean diameter, add the logarithm of the length, and the constant logarithm 7.53148 for wine gallons, or 7.44491* for beer gallons, and the sum rejecting 10 from the index, will be the logarithm of the content. Thus, in the preceding example.

Mean diam. 26.8. log. 1,42813×2=2.85626
Length···· 38. ········· log. ···· 1.57978
Constant log. for wine gallons ···· 7.53148

Content···· 92.79 ······ log. ···· 1.96752

III.

The length of a cask of the second variety, is 29¼ inches, bung diameter 25¾ inches, and head diameter 21½ inches; required its content in wine gallons? Answ. 58½ gallons.

SECTION

---

* These logarithms are those of 231 and 282 subtracted from that of .7854 respectively.

SECTION V.

CONTAINING

THE USE OF THE GAUGING RULE,

IN FINDING THE CONTENT OF A CISTERN, COUCH, OR FLOOR, OF MALT.

In the process of converting barley into malt, the barley is put into a *cistern*, and covered with water; after it has remained a certain time in this state, it is put into a frame called a *couch*-frame; and having continued a proper time, it is laid upon a floor; and is hence called a *floor* of malt. The cistern, and couch are, generally, of a rectangular figure, and the floor of malt is often irregular.

PROBLEM.

To find the content of a cistern, couch, or floor of malt, in bushels.

RULE.

Find the mean length, the mean breadth, and the mean depth of the cistern, couch, or floor of malt. Then, set the length on B. to the depth on the line M. D; and opposite to the breadth on A, is the content in bushels on B.

EXAMPLES.

I.

The length of a cistern is 116 inches, breadth 73, and depth 33; required the content?

Set

floor in the couch

Set 116 on B, to 33 on M. D; and opposite to 73 on A, is the content 130 bushels nearly on B.

II.

Let the length of a couch be 84 inches, breadth 48 inches, and depth 24 inches; required its content?

Set 84 on B. to 24 on M. D; and opposite to 48 on A, is the content 45 bushels on B.

III.

The length of a floor of malt is 180 inches long, 128 inches broad, and 7 inches deep; required its content in malt bushels?

Put 180 on B. to 7 on M.D; then, since the end of the slide is to the right of 128 on A, observe the division on the slide, which is opposite to the right hand end of the line M. D.—Then move the slide until this division is at the other end of the line M.D; and opposite to 128 on A. is the content 75 bushels on B.

REMARKS.

I.

If the right hand end of the slide be to the left of the division on A. representing the breadth, then the division on B. opposite to the left extremity of the line M. D. is to be observed, and the slide is to be moved until this division coincides with the right hand end of the line M. D. and the division on B, opposite to the breadth on A. will be the content in bushels.

II.

The content, in malt bushels, of a vessel in form of a cylinder, is found as follows.

Set the depth on the line B, to the gauge point M. R on D; and opposite to the diameter on D, is the content in bushels on B.

EXAMPLE.

Required the content in malt bushels, of a cylindric vessel, whose depth is 21 inches, and diameter 74 inches?

Set 21 on C. to the gauge point M. R. on D; and opposite to 74 on D, is the content 42 bushels on B.

III.

If the vessel is eliptical, find a geometrical mean between the transverse and conjugate diameters, by Prob. VIII. page 24; with which, and the depth, proceed as before.

CHAP. IV.

CONTAINING

THE DESCRIPTION

OF THE

DIAGONAL OR GAUGING ROD,

AND

ITS USE IN FINDING THE CONTENTS OF CASKS.

SECTION I.

THE DESCRIPTION OF THE GAUGING ROD.

THE most expeditious method of finding the content of a cask, and with a very considerable degree of accuracy, especially when the cask is small, or similar to that to which the rule is adapted; is by the *diagonal rod.*

This rod is generally four feet long, and two fifths of an inch square; and for the convenience of being portable, it is divided into four parts; each a foot long, and connected by brass joints; one end of the rod is sloped, and shielded with brass.

The principal lines upon this rod, and from which it derives its name, are the two diagonal lines, marked *beer* and *wine* gallons; these lines may be constructed as follows.

By experiment, it is found that a cask, whose diagonal line is 30 inches, contains 60 beer gallons; consequently, the division representing 60 gallons is placed at 30 inches. Now, since similar solids are to each other in the triplicate ratio of their homologous sides; therefore, as 60 gallons, is to 27000, the cube of 30 inches; or, which is the same as 1 is to 450, so is any number of gallons, to the cube of the length of the diagonal in inches, of a cask containing the proposed number of gallons. Thus, let it be required to find the length of the diagonal of a cask, containing 36 gallons?

The number 450, multiplied by 36, the product is 16200; the cube root of which is 25.3 inches, the length of the diagonal; therefore, at 25.3 inches, the division representing 36 gallons is to be marked.

By proceeding in this manner, to find the length of the diagonal, for each gallon, within the limits of the length of the rod, and marking off the corresponding divisions, the rod will be graduated. And since the proportion of beer to wine gallons, is as 231 to 282, or as 9 to 11 nearly; therefore, the line of wine gallons is easily deduced from that of beer gallons.

Upon one of the sides of the rod, is a scale of inches, divided into tenths; and a line marked hogs^{d}. St. for ascertaining the ullage of a hogshead, whose axis is perpendicular to the horizon. The following side of the rod contains tables, exhibiting the ullage of lying, or horizontal casks of various capacities, as of 32½ gallons, 63 gallons,

lons, 84, 108, 110, and 120 gallons, the ullage being expressed in whole gallons, and the depth in inches, and tenths of an inch. The fourth side of the rod contains lines for finding the ullage of various casks of known dimensions, as Half Anker, Firkin, Barrel, Half Hogshead, Puncheon, lying or standing. This is performed by putting that end of the rod, from which the divisions answering to the given casks are numbered, into the bung hole, until it rests upon the opposite stave, and the division on the rod intersected by the surface of the fluid will be the ullage.

SECTION II.

THE MANNER OF USING THE GAUGING ROD.

PROBLEM.

To find the content of a cask, by the gauging rod.

RULE.

Put that end of the rod which is covered with brass, into the bung hole of the cask, extend it to the opposite corner, and observe the gallons and parts of a gallon on the rod, at the bung; then turn the rod until its end be at the farthest point at the other head of the cask; and again observe the gallons and parts of a gallon on the

the rod at the bung; if it is the same as before, it will be the content of the cask; if not, half the sum of the divisions will be the content of the cask, in beer, or wine gallons.

EXAMPLE.

A cask is 21 inches long, bung diameter 19 inches, and head diameter 16 inches; required its content by the rod?

By putting the end of the rod into the bung, and extending it to the opposite corner, the division on the line marked beer gallons at the bung, will be about 19, the content required.

The content of the cask may be found independent of the diagonal rod as follows:

From the bung diameter subtract half the difference between it and the head diameter; to the square of which, add the square of half the length of the cask; multiply this sum by its square root, and the product divided by 450, will give the content of the cask in beer gallons.

Thus, in the preceding example, the difference between the bung and head diameters is 3, the half of which is 1.5, which, subtracted from 19, the remainder is 17.5: the square of 17.5 is 306.25; and that of 10.5, half the length of the cask, is 110.25; the sum is 416.5, of which the square root is 20.4. Now, 416.5 multiplied by 20.4, the product is 8496.6; which, divided by

450,

450, the quotient 18.9 nearly, is the content in beer gallons. The reason of this will be obvious from what is said in page 130.

The method of finding the ullage of a cask by this rule has been already fully explained.

CHAP. V.

CONTAINING

THE DESCRIPTION AND USE OF THE ULLAGE RULE.

SECTION I.

THE DESCRIPTION OF THE ULLAGE RULE.

THIS rule is generally a foot or eighteen inches long; it is divided only upon one side, in which are two slides: the first slide contains a double line of numbers, called B; and there is a similar line above upon the stock, called A. The second slide contains two double lines of numbers: between the slides is a line marked *Seg. St.* or Segments standing; and below the slide is a line marked *Seg. Ly.* or Segments lying: the first of these lines is for finding the ullage of a cask whose axis is perpendicular to the horizon, and the other for a cask whose axis is horizontal. All these lines being upon one side of the rule, are seen at one view;

and, therefore, are more convenient for practice, than when placed upon different sides.

The method of constructing the abovementioned lines has been explained at page 116, and the manner of using them is shewn in the following Section.

SECTION II.

CONTAINING

THE USE OF THE ULLAGE RULE,

IN

Ascertaining the Quantity of Liquor in a Cask.

When a cask is partly full, the quantity of liquor in it is called the *ullage*. In finding the ullage of a cask, it is supposed to be so placed, that its axis is either parallel, or perpendicular to the horizon. In the first case, the cask is said to be *lying*, and in the second *standing*. The depth of liquor in a cask at the bung, when lying, or at the head, when standing, is called the *wet inches*; and the remainder is called the *dry inches*.

PROBLEM I.

To find the ullage of a Cask whose axis is parallel to the horizon.

RULE.

Set the bung diameter upon the slide, to 100 upon the line marked Seg. Ly.; then opposite to the wet inches on the slide, is a certain quantity on the line Seg. Ly, Now, set 100 on B, to the content of the cask on A; and opposite to the above found quantity on B, is the ullage of the cask on A,

EXAMPLE.

A cask, lying horizontal, whose content is 72 gallons, has 22 wet inches; and the bung diameter is 30 inches; required the quantity of liquor in the cask?

Set the bung diameter 30 on the slide, to 100 on the line S. L.; then opposite to the wet inches 22 on the slide, is 80 on S. L.—Now, set 100 on B, to 72, the content of the cask on A; and opposite to 80 on B, is 57 and a fraction on A, the ullage required.

REMARK.

The ullage may be found independent of the rule, as follows.

Divide the wet inches by the bung diameter; and increase, or diminish this quotient by one-fourth of the difference between it and .5, according as it is more or less than .5; and the sum, or remainder, being multiplied by the content of the cask, will give the ullage, or quantity of liquor in it. Thus—

The wet inches 22, divided by 30, the bung diameter, the quotient is .733; now, this quotient being increased by .058, one-fourth of the difference between it and .5; the sum .791 multiplied by 72, the product is 57 gallons nearly, the required ullage.

REMARK.

Another method of finding the ullage of a lying cask, is by means of a table of segments of a circle, whose area is 1; and the diameter, 1·12837 is divided into 100 equal parts.—These parts are titled V, or versed sine, opposite to which are the areas of the corresponding segments. The method of constructing this table is obvious; and the manner of using it is as follows:

Annex two cyphers to the wet inches, and divide this sum by the bung diameter; find the quotient in the table under V, and take out the corresponding number, which, multiplied by the content of the cask, the product will be the ullage.

The line M. upon Mr. B. Martin's mathematical rule, is the semi-diameter divided into 50 equal parts, and the numbers on the line N. are the areas of the corresponding segments.

PROBLEM II.

To find the ullage of a Cask whose axis is perpendicular to the horizon.

RULE.

Set the length of the cask on C. to 100 on the line Seg. St.; and opposite to the wet inches on C. is a certain quantity on Seg. St. Now, set 100 on B. to the content of the cask on A; and opposite to the above found quantity on B. is the ullage on A.

EXAMPLE.

It is required to find the ullage of a cask standing, in wine gallons, its length being 35 inches, bung and head diameters 26 and 22 respectively; the wet inches being 16 inches, and the content of the cask 72 wine gallons?

Set 35, the length of the cask, on C. to 100 on D; and opposite to 16, the wet inches on C. is about 45 on D. Now, set 100 on B. to 72, the content of the cask, on A; and opposite to 45, the above found number, on B. is the ullage 32½ gallons nearly on A.

REMARK.

The ullage may be found without the rule, as follows:

Divide the wet inches by the length of the cask; increase the quotient by one-tenth of the difference between it and .5, if it exceeds that quantity, but diminish it if less; then, the sum, or remainder, multiplied by the content of the cask, will give the ullage. Thus, in the preceding example—

The wet inches 16, being divided by 35, the length of the cask, the quotient is .457, which being less than .5; one-tenth of the difference between it and .5, viz. .004, being deducted from the quotient .457, and the remainder .453, multiplied by 72, the content of the cask, the product 32.6 is the ullage required.

FINIS.

PROPOSALS for PUBLISHING by SUBSCRIPTION,

BY

A. MACKAY, LL.D. F.R.S. EDIN. &c.

I.

A NEW EDITION, (being the Third) much improved and enlarged,

OF

THE THEORY AND PRACTICE

OF FINDING

THE LONGITUDE AT SEA OR LAND:

To which are added,

Various Methods of determining the Latitude of a Place,

AND

VARIATION OF THE COMPASS;

WITH

NEW TABLES,

IN TWO VOLUMES OCTAVO,

Price to Subscribers One Guinea and a Half in Boards.—To Non-Subscribers the Price will be advanced.

The following is an Abstract of the Contents of this Work.

Of the Figure and Magnitude of the Earth; Definitions, Principles, &c.; of the Length of the Year; of the Fixed Stars, Planets, Moon, &c.

Of Hadley's Quadrant, the Sextant, aud Circular Instrument; the Manner of taking a complete Set of Lunar Observations; of the Corrections to be applied to the Altitude of an Object observed at Sea, and to the observed Distance between two Objects.

The Method of finding the Longitude by Lunar Observations; Historical Introduction; Preparatory Problems; Various Methods of ascertaining Time, and regulating a Chronometer, or Watch, at Sea or Land; of the Methods of clearing the Apparent Distance between the Moon the Sun, or a fixed Star, from the Effects of Refraction and Parallax. Of finding the Longitude at Sea or Land.—1st. By Observations of the Distance between, and Altitudes of the Sun and the Moon.—2d. By an Observation of the Distance only.—3d. New Method of finding the Longitude, Latitude, Apparent Time, &c. from the same Set of Observations.—4th. Of finding the Longitude by an Observation of the Distance between the Moon and a Star not used in the Nautical Almanack.—5th. By an Observation of the Distance between the Moon and a Planet.—6th. By the Moon's Transit over the Meridian.—7th. By the Meridian Altitude of the Moon.—8th. By the Moon's Altitude observed at any Time.—9th. By an Eclipse of the Moon.—10th. Of the Sun.—11th. By Occultations of fixed Stars by the Moon.—12th. By the Eclipses of the Satellites of Jupiter.—13th. By a Chronometer.—14th. And by the Variation Chart.

The Demonstrations of the preceding Methods.

The Method of finding the Latitude by the Meridian Altitude of the Sun, Moon, fixed Stars, and Planets; by equal Altitudes: by double Altitudes of the Sun; by double Altitudes of the Moon; by double Altitudes of the fixed Stars; by three Altitudes of the Sun; and without the Sun's Altitude.

Of finding the Variation of the Compass, by an Observation of the Sun's Amplitude; by an Azimuth; by equal Altitudes of the Sun; with various other Methods of finding the Variation.

The second Volume contains seventy Tables, with their Explanation prefixed; and an Appendix, containing two Tables; the Method of finding the Time of the rising and setting of the Sun, Moon, and fixed Stars; the Time of the Beginning, End, and Duration of Twilight, &c.

The following are Extracts from some of the Reviews, relative to the first Edition of this Work.

"From the preceding abstract of the contents of this valuable work, our readers will be better able to judge what information they may derive from it, than they could have been by any detached extract which we might have selected. Of the Author's extensive and accurate acquaintance with the subject to which he has directed our attention, none who peruse what he has written can entertain any doubt. To the Navigator and Astronomer, and also to the practical Geographer and Surveyor, this work will be instructive and useful. We cannot conclude without expressing our wishes, that the Author may be suitably encouraged in the prosecution of the plan which he has undertaken, of publishing a distinct Treatise on the Theory and Practice of Navigation, and another in three volumes on the Theory and Practice of Astronomy."—
Monthly Review for March, 1794.

"Throughout the articles which we have specified, Dr. **Mackay's** zeal and ability to instruct his readers are equally manifest. He has not only written fully on the subjects themselves, but has also introduced such collateral matter as was likely to satisfy the inquisitive reader, and illustrate his principal design. In his description of instruments, we meet with the minuteness of an artist; in his application of them, we find the attention of an accurate observer; and in the demonstration of his rules, we see the precision of a practised geometrician."—*British Critic for March,* 1794.

"A very useful praxis and investigation of the various modes of finding the Longitude. The Author first gives a concise account of the planetary system: then describes the various instruments used in taking altitudes; and after a sufficient number of preparatory problems, the mode of finding the Longitude by lunar observations; eclipses of the Sun and Moon; occultations of fixed stars by the Moon; eclipses of the Satellites of Jupiter; by the chronometer; and the variation charts. The praxis is in the former; the demonstrations in the latter part of the first volume, the second volume contains the necessary tables. After each rule is a sufficient number of examples to give a perfect knowledge of the use of it. The Navigator who has mastered the problems in this work, will not, with a clear sky over his head, find himself at a loss for his reckoning; and it might be made a useful compendium in a long voyage; for, by daily perusal, the younger proficients in the art of Navigation may acquire a taste for a mode of observation, which, we fear, notwithstanding its evident utility, has by no means obtained general practice. In speaking this of younger proficients, we do not mean to say, that any person, whether at land or at sea, who employs himself in finding the Longitude of the place he is in, will not reap much advantage by having the rules and examples laid down in this work to guide him in his practice; for no method will easily occur, of which he will not find here an example."—*Critical Review for August,* 1794.

"The Theory and Practice of finding the Longitude, &c. is a work which will be found of considerable utility and importance to the Navigator and to the Astronomer, as the Author appears to be accurately acquainted with the subject on which he treats, and is perspicuous and satisfactory in his methods of explanation."—*New Annual Register for* 1793.

"This publication (the Author's Treatise on Navigation) and that on the Longitude by the same author, form the most correct and practical system of Navigation and Nautical Science hitherto published in this country: they may be considered not only of individual utility, but of national importance."—*Anti-Jacobin Review for September*, 1804.

See also L'Esprit des Journeaux for April, 1794—Bibliographie Astronomique, &c.

II.

ELEMENTARY PRINCIPLES
OF
GEOGRAPHY,
WITH
THE USE OF THE GLOBES.

Price to Subscribers Seven Shillings, in boards.—To Non-Subscribers the Price will be advanced.

Those who wish to subscribe for either of the above Works, are requested to intimate the same to the Author, No. 2, *George-Street, Trinity-Square, London.*

Where may be had,

	£.	s.	d.
His Treatise on the Theory and Practice of finding the Longitude at Sea or Land, in Two Volumes, 8vo. in Boards; second edition	1	1	0
Complete Navigator	0	10	6

"From Dr. Mackay's well-earned fame as a mathematician and astronomer, and from the specimens which he has given of his practical knowledge of nautical science, we, of course, expected that a regular Treatise of Navigation from his pen would be a valuable acquisition to seamen, and the work before us fully answers our expectations. It is a clear, well digested, and masterly performance, containing, besides what is useful in other publications, much new and important matter."—
Anti-Jacobin Review, for September, 1804.

"This is evidently the work of a man of science, of one who understands the subject which he professes to teach. To us there appears to be nothing wanting for the complete instruction of the young mariner in nautical affairs. The elementary principles are laid down with clearness, and with as much conciseness as is consistent with perspicuity. The structure and use of Hadley's quadrant, and other instruments, in the third book, will be readily understood by Dr. Mackay's account of them. The ship's journal from England to Madeira, is admirably calculated to shew how much practical seamanship depends on theory; and the miscellaneous articles which include the application of trigonometry to mensuration; the method of surveying coasts and harbours, of knowing the fixed stars; the nature of the winds, and when and where they may be expected; and the rules for predicting the weather; will be all of considerable use to the mariner."—

Imperial Review, for August, 1804.

	£.	s.	d.
Collection of Mathematical Tables …………	0	7	0
Method of finding the Longitude by Lunar Observations …………	0	5	0
Various Methods of finding the Latitude of a Ship at Sea, and the Variation of the Compass ……	0	2	6
Description and Use of the Sliding Gunter in Navigation …………	0	4	6
Description and Use of the Sliding Rule, in Arithmetic, and in the Mensuration of Surfaces and Solids; The Description and Use of the Ship Carpenter's Rule in Mensuration, and in making Masts, Yards, Capstan-barrels, &c. with an Investigation of the Method of constructing the Eight, Ten, and Twelve Square Lines; The Description and Use of the Gauging Rule, the Diagonal Rod, and Ullage Rule; second edition …………	0	3	6
The Commencement of the Nineteenth Century determined upon unerring Principles ………	0	1	6
A Table, exhibiting the Moon's Age, by Inspection, from the Year 1800 to 1894 inclusive …	0	0	6